Appendix Roadmap

This appendix contains five sections. Appendix A reports the Nash equilibria for Bertrand competition games with homogeneous customers, asymmetric capacities and differentiated qualities. Appendix B presents two major proofs under low demand, demonstrating the mechanisms at play throughout the paper. Appendix C connects the pooled capacity and pro rata capacity allocation cases under discriminatory pricing, justifying our focus in the paper on pro rata capacity allocation. Appendix D establishes the robustness of our findings under moderate demand. Appendix E includes a sensitivity analysis for the parameter α. Collectively, this appendix paints a complete picture of Bertrand competition games under heterogenous customers and endogenous inventory allocation. The Electronic Companion includes remaining proofs and extensions.

Appendix A: Full Equilibrium Characterization for Homogeneous Markets

Table 2 provides a full characterization of the Nash equilibrium for the Bertrand competition game in homogeneous markets under asymmetric capacities and differentiated qualities. Regions U0 to U6 define cases with homogenous QSC (i.e., $D=D_u,\ D_i=D_i^u$, and $I_i^u=I_i$ for $i\in\{1,2\}$), whereas Regions P0 to P5 define cases with homogenous PSC (i.e., $D=D_p,\ D_i=D_i^p$, and $I_i^p=I_i$ for $i\in\{1,2\}$). Propositions 1 and 2 are two special cases of Table 2, where the firms have the same capacities. For conciseness, we focus on Regions U0 to U6 (P0 to P5 are special cases with $\Delta=0$) and relegate all the proofs to EC.1.

Table 2 **Equilibrium profits under quality differentiation and asymmetric capacities. We define 13 regions, seven with QSC (U0 to U6) and six with PSC (P0 to P5).**

Region	$\overline{\Pi}_1$	$\overline{\Pi}_2$	Definition
U0	$I_1 p_{\max}$	$I_2 p_{\max}$	$D\ge I_1+I_2$
U1	0	$D\Delta$	$D<\min\left\{\frac{1}{1-\frac{\Delta}{p_{\max}}}I_1, I_2\right\}$
U2	$(D-I_2)p_{\max}$	$I_2 p_{\max}$	$D>\left(1-\frac{\Delta}{p_{\max}}\right)I_1+I_2$ or $\frac{p_{\max}}{\Delta}I_2\le D<I_1$
U3	$(D-I_2)p_{\max}$	$I_2\left(\frac{D-I_2}{I_1}p_{\max}+\Delta\right)$	$I_1<I_2\le D$ and $I_1+I_2-\frac{\Delta}{p_{\max}}\frac{I_1I_2}{I_2-I_1}\le D<\left(1-\frac{\Delta}{p_{\max}}\right)I_1+I_2$; or $I_2<I_1\le D<\left(1-\frac{\Delta}{p_{\max}}\right)I_1+I_2$
U4	$(D-I_2)p_{\max}$	$I_2\left(\frac{D-I_2}{D}p_{\max}+\Delta\right)$	$I_2<D<\min\left\{I_1,\frac{p_{\max}}{\Delta}I_2\right\}$
U5	$I_1\left(\frac{D-I_1}{I_2}p_{\max}-\Delta\right)$	$(D-I_1)p_{\max}$	$I_1<I_2\le D<I_1+I_2-\frac{\Delta}{p_{\max}}\frac{I_1I_2}{I_2-I_1}$
U6	$I_1\left(\frac{D-I_1}{D}p_{\max}-\Delta\right)$	$(D-I_1)p_{\max}$	$\frac{1}{1-\frac{\Delta}{p_{\max}}}I_1<D<I_2$
P0	$I_1 p_{\max}$	$I_2 p_{\max}$	$D\ge I_1+I_2$
P1	0	0	$D\le\min\{I_1,I_2\}$
P2	$(D-I_2)p_{\max}$	$\frac{I_2}{D}(D-I_2)p_{\max}$	$I_2\le D\le I_1$
P3	$\frac{I_1}{D}(D-I_1)p_{\max}$	$(D-I_1)p_{\max}$	$I_1\le D\le I_2$
P4	$(D-I_2)p_{\max}$	$\frac{I_2}{I_1}(D-I_2)p_{\max}$	$I_2\le I_1\le D$
P5	$\frac{I_1}{I_2}(D-I_1)p_{\max}$	$(D-I_1)p_{\max}$	$I_1\le I_2\le D$

U0: $D\ge I_1+I_2$**.** The Nash equilibrium is $(p_1,p_2)=(p_{\max},p_{\max})$, so $\overline{\Pi}_1=I_1p_{\max}$ and $\overline{\Pi}_2=I_2p_{\max}$.

U1: $D\le\min\{\frac{1}{1-\frac{\Delta}{p_{\max}}}I_1,I_2\}$**.** We divide the analysis into three cases.

– If $D<\min\{I_1,I_2\}$. This is the uncapacitated case, so the Nash equilibrium is the same as in Section 3.2:

$$\begin{aligned} Q_{F_1}(0)&=0,\quad F_1(p_1)\ge 1-\frac{\Delta}{p_1+\Delta},\quad \text{for all } p_1\le p_{\max}-\Delta,\quad Q_{F_2}(\Delta)=1,\\ \overline{\Pi}_1&=0,\quad \overline{\Pi}_2=D\Delta. \end{aligned}$$

– If $I_1 < D < I_2, D < 2I_1$, we have

$$\begin{aligned} \pi_1(p_1, F_2) &= p_1\left[I_1 - I_1F_2(p_1+\Delta) + \tfrac{D}{2}Q_{F_2}(p_1+\Delta)\right], \\ \pi_2(p_2, F_1) &= p_2\left[D - I_1F_1(p_2-\Delta) + (I_1 - \tfrac{D}{2})Q_{F_1}(p_2-\Delta)\right]. \end{aligned}$$

The Nash equilibrium is characterized by:

$$\begin{aligned} Q_{F_1}(0) &= 0, \quad F_1(p_1) \geq \tfrac{D}{I_1}\left(1 - \tfrac{\Delta}{p_1+\Delta}\right), \quad \text{for all } p_1 \leq p_{\max} - \Delta, \\ Q_{F_2}(\Delta) &= 1, \\ \overline{\Pi}_1 &= 0, \quad \overline{\Pi}_2 = D\Delta. \end{aligned}$$

– If $2I_1 \leq D < I_2$, we have

$$\begin{aligned} \pi_1(p_1, F_2) &= p_1[I_1 - I_1F_2(p_1+\Delta) + I_1Q_{F_2}(p_1+\Delta)], \\ \pi_2(p_2, F_1) &= p_2[D - I_1F_1(p_2-\Delta)]. \end{aligned}$$

The Nash equilibrium is once again characterized by:

$$\begin{aligned} Q_{F_1}(0) &= 0, \quad F_1(p_1) \geq \tfrac{D}{I_1}\left(1 - \tfrac{\Delta}{p_1+\Delta}\right), \quad \text{for all } p_1 \leq p_{\max} - \Delta, \\ Q_{F_2}(\Delta) &= 1, \\ \overline{\Pi}_1 &= 0, \quad \overline{\Pi}_2 = D\Delta. \end{aligned}$$

Note that whether or not $2I_1 < D$ (or $2I_2 < D$) does not affect the equilibrium profits, because the mixed strategies have at most one atom at $p_{\max}$, and the demand sharing at $p_1 + \Delta = p_2$ does not affect the equilibrium profits. We thus thereafter omit the distinction between $2I_1$(or $2I_2$) is larger than D, so we just assume that whenever $I_1 < D$ (or $I_2 < D$), we have $2I_1 > D$ (or $2I_2 > D$).

U2: $D \geq \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$ **or** $\frac{p_{\max}}{\Delta} I_2 \leq D < I_1$**.**

– If $D \geq \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$, then $(D - I_2)p_{\max} \leq I_1(p_{\max} - \Delta)$, meaning that Firm 1 is always better off playing $p_{\max}$. Consequently, Firm 2 will also play $p_{\max}$.

– If $\frac{p_{\max}}{\Delta} I_2 \leq D < I_1$, then $(D - I_2)p_{\max} \leq D(p_{\max} - \Delta)$, meaning that Firm 1 is always better off playing $p_{\max}$. Consequently, Firm 2 will also play $p_{\max}$.

U3: $I_1 < I_2 \leq D < \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$ **and** $\frac{\Delta}{p_{\max}} \geq \frac{(I_1+I_2-D)(I_2-I_1)}{I_1I_2}$**; or** $I_2 < I_1 \leq D < \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$**.**

– If $I_1 < I_2 \leq D < \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$ and $\frac{\Delta}{p_{\max}} \geq \frac{(I_1+I_2-D)(I_2-I_1)}{I_1I_2}$, we have:

$$\begin{aligned} \pi_1(p_1, F_2) &= p_1\left[I_1 - (I_1+I_2-D)F_2(p_1+\Delta) + (I_2 - \tfrac{D}{2})Q_{F_2}(p_1+\Delta)\right], \\ \pi_2(p_2, F_1) &= p_2\left[I_2 - (I_1+I_2-D)F_1(p_2-\Delta) + (I_1 - \tfrac{D}{2})Q_{F_1}(p_2-\Delta)\right]. \end{aligned}$$

The Nash equilibrium can then be characterized as follows:

$$\begin{aligned} F_1(p_1) &= \frac{I_2 - \frac{I_2\left(\frac{D-I_2}{I_1}p_{\max}+\Delta\right)}{p_1+\Delta}}{I_1+I_2-D}, p_1 \in \left[\tfrac{D-I_2}{I_1}p_{\max}, p_{\max}-\Delta\right], \quad Q_{F_1}(p_{\max}) = \frac{I_1 - D + \frac{I_2\left(\frac{D-I_2}{I_1}p_{\max}+\Delta\right)}{p_{\max}}}{I_1+I_2-D}, \\ F_2(p_2) &= \frac{I_1 - \frac{(D-I_2)p_{\max}}{p_2-\Delta}}{I_1+I_2-D}, p_2 \in \left[\tfrac{D-I_2}{I_1}p_{\max}+\Delta, p_{\max}\right), \quad Q_{F_2}(p_{\max}) = \frac{I_2 - D + \frac{(D-I_2)p_{\max}}{p_{\max}-\Delta}}{I_1+I_2-D}, \\ \overline{\Pi}_1 &= (D-I_2)p_{\max}, \quad \overline{\Pi}_2 = I_2\left(\tfrac{D-I_2}{I_1}p_{\max}+\Delta\right). \end{aligned}$$

– If $I_2 < I_1 \leq D < \left(1 - \frac{\Delta}{p_{\max}}\right) I_1 + I_2$, the profit functions are the same as before. Moreover, since $I_2 < I_1$, $\frac{\Delta}{p_{\max}} \geq \frac{(I_1+I_2-D)(I_2-I_1)}{I_1I_2}$, so that the previous mixed strategy still characterizes a Nash equilibrium.

U4: $I_2 < D < \min\{I_1, \frac{p_{\max}}{\Delta} I_2\}$**.** We have

$$\begin{aligned}\pi_1(p_1,F_2) &= p_1\left[D - DF_2(p_1+\Delta) + \tfrac{D}{2}Q_{F_2}(p_1+\Delta)\right],\\ \pi_2(p_2,F_1) &= p_2\left[I_2 - I_2F_1(p_2-\Delta) + \tfrac{D}{2}Q_{F_1}(p_2-\Delta)\right].\end{aligned}$$

The Nash equilibrium is given by:

$$\begin{aligned}F_1(p_1) &= 1 - \frac{\frac{D-I_2}{D}p_{\max}+\Delta}{p_1+\Delta}, p_1 \in \left[\tfrac{D-I_2}{D}p_{\max}, p_{\max}-\Delta\right], \quad Q_{F_1}(p_{\max}) = \frac{D-I_2}{D} + \frac{\Delta}{p_{\max}},\\ F_2(p_2) &= \frac{D - \frac{(D-I_2)p_{\max}}{p_2-\Delta}}{I_2}, p_2 \in \left[\tfrac{D-I_2}{D}p_{\max}+\Delta, p_{\max}\right), \quad Q_{F_2}(p_{\max}) = \frac{I_2 - D + \frac{(D-I_2)p_{\max}}{p_{\max}-\Delta}}{I_2},\\ \overline{\Pi}_1 &= (D-I_2)p_{\max}, \quad \overline{\Pi}_2 = I_2\left(\tfrac{D-I_2}{D}p_{\max}+\Delta\right).\end{aligned}$$

U5: $I_1 < I_2 \le D < I_1 + I_2 - \frac{\Delta}{p_{\max}}\frac{I_1 I_2}{I_2 - I_1}$**.** This is equivalent to $\frac{\Delta}{p_{\max}} < \frac{(I_1+I_2-D)(I_2-I_1)}{I_1 I_2}$. We have

$$\begin{aligned}\pi_1(p_1,F_2) &= p_1\left[I_1 - (I_1+I_2-D)F_2(p_1+\Delta) + \left(I_2 - \tfrac{D}{2}\right)Q_{F_2}(p_1+\Delta)\right],\\ \pi_2(p_2,F_1) &= p_2\left[I_2 - (I_1+I_2-D)F_1(p_2-\Delta) + \left(I_1 - \tfrac{D}{2}\right)Q_{F_1}(p_2-\Delta)\right].\end{aligned}$$

The Nash equilibrium is given by:

$$\begin{aligned}F_1(p_1) &= \frac{I_2 - \frac{(D-I_1)p_{\max}}{p_1+\Delta}}{I_1+I_2-D}, p_1 \in \left[\frac{D-I_1}{I_2}p_{\max}-\Delta, p_{\max}-\Delta\right],\\ F_2(p_2) &= \frac{I_1 - \frac{I_1\left(\frac{D-I_1}{I_2}p_{\max}-\Delta\right)}{p_2-\Delta}}{I_1+I_2-D}, p_2 \in \left[\frac{D-I_1}{I_2}p_{\max}, p_{\max}\right), \quad Q_{F_2}(p_{\max}) = \frac{I_2 - D + \frac{I_1\left(\frac{D-I_1}{I_2}p_{\max}-\Delta\right)}{p_{\max}-\Delta}}{I_1+I_2-D},\\ \overline{\Pi}_1 &= I_1\left(\frac{D-I_1}{I_2}p_{\max}-\Delta\right), \quad \overline{\Pi}_2 = (D-I_1)p_{\max}.\end{aligned}$$

U6: $\frac{1}{1-\frac{\Delta}{p_{\max}}} I_1 < D < I_2$**.** The profit functions are the same as that in U1 with $I_1 < D < \min(I_2, 2I_1)$:

$$\begin{aligned}\pi_1(p_1,F_2) &= p_1\left[I_1 - I_1F_2(p_1+\Delta) + \tfrac{D}{2}Q_{F_2}(p_1+\Delta)\right],\\ \pi_2(p_2,F_1) &= p_2\left[D - I_1F_1(p_2-\Delta) + \left(I_1 - \tfrac{D}{2}\right)Q_{F_1}(p_2-\Delta)\right].\end{aligned}$$

The Nash equilibrium is given by:

$$\begin{aligned}F_1(p_1) &= \frac{D - \frac{(D-I_1)p_{\max}}{p_1+\Delta}}{I_1}, p_1 \in \left[\tfrac{D-I_1}{D}p_{\max}-\Delta, p_{\max}-\Delta\right],\\ F_2(p_2) &= 1 - \frac{\frac{D-I_1}{D}p_{\max}-\Delta}{p_2-\Delta}, p_2 \in \left[\tfrac{D-I_1}{D}p_{\max}, p_{\max}\right), \quad Q_{F_2}(p_{\max}) = \frac{\frac{D-I_1}{D}p_{\max}-\Delta}{p_{\max}-\Delta},\\ \overline{\Pi}_1 &= I_1\left(\tfrac{D-I_1}{D}p_{\max}-\Delta\right), \quad \overline{\Pi}_2 = (D-I_1)p_{\max}.\end{aligned}$$

Appendix B: Proofs of Main Results Under Low Demand

In this appendix, we prove Proposition 3, which characterizes the Nash equilibrium under uniform pricing, and Proposition 7, which characterizes the Nash equilibrium under discriminatory pricing. These two proofs demonstrate the mechanisms to prove the most results in this paper. All other proofs are relegated to Electronic Companion. For notational convenience, let $a = \frac{\Delta}{p_{\max}} \le \frac{2}{5}$.

B.1. Proof of Proposition 3

We first establish in Lemma 1 (proved in EC.3) that the nonlinear system of Equations (1)–(2) admits a unique positive (real) solution and then show that each firm has no incentive for unilateral deviations.

Lemma 1. *The system of Equations* (1)–(2) *admits a unique positive (real) solution, such that:*

$$\hat{p} < \frac{D_u}{D_p}\Delta, \tag{3}$$

$$\tilde{p} < \frac{D_p}{D_u}\Delta, \tag{4}$$

$$\hat{p} \le \left(1 + \frac{D_p}{D_u}\right)\Delta, \tag{5}$$

$$\tilde{p} \le \frac{D_u}{D_p}\Delta. \tag{6}$$

Proof that the solution given in Proposition 3 is a Nash equilibrium. We have:

$$\begin{aligned}
\tilde{p} < \hat{p} &\iff \frac{1}{\tilde{p}+\Delta} > \frac{1}{\hat{p}+\Delta}, \\
&\iff \frac{1}{\hat{p}} - \frac{1}{\hat{p}+\Delta}\frac{D_p}{D_u} > \frac{1}{\hat{p}+\Delta} \quad \text{from Equation (1),} \\
&\iff \hat{p} < \frac{D_u}{D_p}\Delta, \quad \text{which is satisfied from Equation (3).}
\end{aligned}$$

Likewise, $\hat{p} < \tilde{p}+\Delta$ is guaranteed by Equation (4). Equations (1)–(2) guarantee the continuity of $F_1(p)$ in $\hat{p}$, and the continuity of $F_2(p)$ in $\tilde{p}$. We next show that there is no incentive for each player to deviate from their supports, by showing that all the prices in the support of F_1 (resp. F_2) yield an expected profit of $\hat{p}D_p$ (resp. $(\tilde{p}+\Delta)D_u$), and that all the other prices yield a lower expected profit.

- For Firm 1:
  - $p_1 \in [\tilde{p},\hat{p}]$: $\pi_1(p_1,F_2) = p_1\{D - D_uF_2(p_1+\Delta) - D_pF_2(p_1)\} = \hat{p}D_p$.
  - $p_1 \in [\hat{p},\tilde{p}+\Delta]$: $\pi_1(p_1,F_2) = p_1\{D - D_uF_2(p_1+\Delta) - D_pF_2(p_1)\} = \hat{p}D_p$.
  - $p \le \hat{p}-\Delta$: $\pi_1(p_1,F_2) = p_1D \le (\hat{p}-\Delta)D$. Note that: $(\hat{p}-\Delta)D \le \hat{p}D_p \iff \hat{p} \le \left(1+\frac{D_p}{D_u}\right)\Delta$, which is satisfied from Equation (5). Therefore, $\pi_1(p_1,F_2) \le \hat{p}D_p$.
  - $\hat{p}-\Delta < p_1 < \tilde{p}$: $\pi_1(p_1,F_2) = p_1\left[D - D_u\left(1-\frac{\hat{p}}{p_1+\Delta}\right)\right] = p_1D_p + \hat{p}D_u\frac{p_1}{p_1+\Delta}$. Note that $p_1D_p + \hat{p}D_u\frac{p_1}{p_1+\Delta}$ is increasing in p_1. Therefore: $\pi_1(p_1,F_2) \le \pi_1(\tilde{p},F_2) = \hat{p}D_p$.
  - $\tilde{p}+\Delta < p_1 < \hat{p}+\Delta$: $\pi_1(p_1,F_2) = p_1\left[D - D_u - D_p\frac{D - \frac{D_p\hat{p}}{p_1-\Delta}}{D_u}\right] = \frac{D_p^2}{D_u}\left[\hat{p}\left(1+\frac{\Delta}{p_1-\Delta}\right) - p_1\right]$. Note that $\frac{D_p^2}{D_u}\left[\hat{p}\left(1+\frac{\Delta}{p_1-\Delta}\right) - p_1\right]$ is decreasing in p_1. Therefore: $\pi_1(p_1,F_2) \le \pi_1(\tilde{p}+\Delta,F_2) = \hat{p}D_p$.
  - $p \ge \hat{p}+\Delta$: $\pi_1(p,F_2) = 0 < \hat{p}D_p$.

- For Firm 2: We repeat the same steps to check Firm 2's profit function in each of the six intervals, – $p_2 \in [\hat{p},\tilde{p}+\Delta]$, – $p_2 \in [\tilde{p}+\Delta,\hat{p}+\Delta]$, – $p_2 \le \tilde{p}$, – $\tilde{p} < p_2 < \hat{p}$, (i) $\hat{p}+\Delta < p_2 < \tilde{p}+2\Delta$, – $p_2 \ge \tilde{p}+2\Delta$, and there is no incentive for Firm 2 to deviate.

Hence, the solution given in Proposition 3 is a Nash equilibrium, and the expected profits are given by: $\overline{\Pi}_1 = \hat{p}D_p$ and $\overline{\Pi}_2 = (\tilde{p}+\Delta)D_u$. □

To show the uniqueness of the Nash equilibrium, we make use of the following lemma (proved in EC.3), which characterizes the supports of NE. Lemma 2 is one of the most technical results in this paper and relies on a series of auxiliary results elicited in EC.3 (Lemmas EC.8–EC.15 and Corollaries EC.1 and EC.2).

LEMMA 2. *Both firms play mixed strategies with no atom and with supports given as: Supp*$(F_1) = [\tilde{p},\tilde{p}+\Delta]$ *and Supp*$(F_2) = [\hat{p},\hat{p}+\Delta]$.

Proof that the solution given in Proposition 3 is the unique Nash equilibrium. It only remains to show that the structure of the Nash equilibrium stated in Proposition 3 is necessary. Since $\hat{p} \in \mathrm{Supp}(F_1)$ and $\tilde{p}+\Delta \in \mathrm{Supp}(F_2)$, we have:

$$\begin{aligned}
\overline{\Pi}_1 &= \pi_1(\hat{p},F_2) = \hat{p}[D - D_uF_2(\hat{p}+\Delta) - D_pF_2(\hat{p})] = \hat{p}D_p, \\
\overline{\Pi}_2 &= \pi_2(\tilde{p}+\Delta,F_1) = (\tilde{p}+\Delta)[D - D_uF_1(\tilde{p}) - D_pF_1(\tilde{p}+\Delta)] = (\tilde{p}+\Delta)D_u.
\end{aligned}$$

- For each $p \in [\tilde{p},\hat{p}]$: $\overline{\Pi}_1 = \pi_1(p,F_2) = p_1[D - D_uF_2(p+\Delta)]$. It comes: $F_2(p+\Delta) = \frac{D - \frac{\overline{\Pi}_1}{p}}{D_u}$.

– For each $p \in [\hat{p}, \tilde{p}+\Delta]$:

$$\begin{aligned}\overline{\Pi}_1 &= \pi_1(p, F_2) = p[D - D_u - D_p F_2(p)],\\ \overline{\Pi}_2 &= \pi_2(p, F_2) = p[D - D_p F_1(p)].\end{aligned}$$

We thus have $F_2(p) = 1 - \frac{\overline{\Pi}_1}{pD_p}$ and $F_1(p) = \frac{D - \frac{\overline{\Pi}_2}{p}}{D_p}$.

– For each $p \in [\tilde{p}+\Delta, \hat{p}+\Delta]$: $\overline{\Pi}_2 = \pi_2(p, F_1) = p[D - D_u F_1(p-\Delta) - D_p]$. It comes: $F_1(p-\Delta) = 1 - \frac{\overline{\Pi}_2}{pD_u}$.

This completes the proof. □

B.2. Proof of Proposition 7

For ease of exposition, we restrict to the case where $D = I = I_1^u + I_1^p = I_2^u + I_2^p$ (the general case where $D < I$ is addressed in Remark 1 below). We consider the "U-regions" and the "P-regions" defined in Table 2. When (I_1^u, I_2^u) falls into a certain U-region, $(I_1^p, I_2^p) = (I - I_1^u, I - I_2^u)$ falls into a certain P-region accordingly. Figure 11 represents the regions in a two-dimensional plane (I_1^u, I_2^u) (D_p, D_u, and $\frac{\Delta}{p_{\max}}$ are given). The solid black lines define the six U-regions, and the dashed lines define the five P-regions.

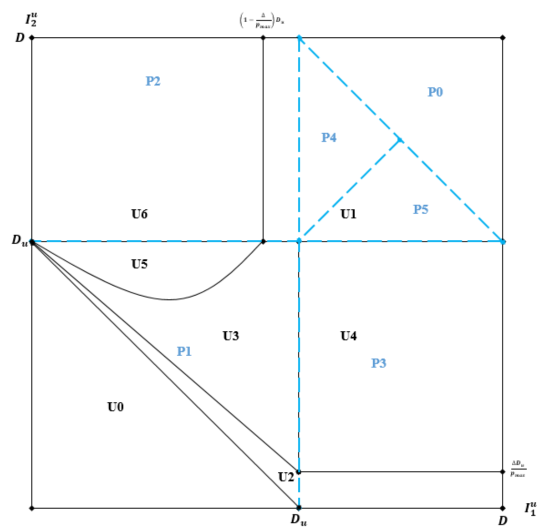


Figure 11 **Illustration of the regions when** $D = I$.

Let us start with an important fact: the horizontal line $I_2^u = \frac{1+a}{2} D_u$ will never intersect Region U5. To see this, note that U5's lower bound is defined by the hyperbola $\frac{D_u - I_1^u}{I_2^u} - \frac{D_u - I_2^u}{I_1^u} = a$. To seek the lowest point of this hyperbola, we take the derivative with respect to I_1^u and set $\frac{\mathrm{d}I_2^u}{\mathrm{d}I_1^u} = 0$:

$$\frac{-I_2^u - (D_u - I_1^u)\frac{\mathrm{d}I_2^u}{\mathrm{d}I_1^u}}{(I_2^u)^2} - \frac{\frac{-\mathrm{d}I_2^u}{\mathrm{d}I_1^u} - (D_u - I_2^u)}{(I_1^u)^2} = 0 \implies \frac{D_u - I_2^u}{(I_1^u)^2} - \frac{1}{I_2^u} = 0.$$

Combining the above with $\frac{D_u - I_1^u}{I_2^u} - \frac{D_u - I_2^u}{I_1^u} = a$, we obtain the expression of the lowest point:

$$\begin{cases} I_1^u = \frac{D_u - aI_2^u}{2}, \\ I_2^u = \frac{D_u}{a+2-2\sqrt{a}}. \end{cases}$$

It can now easily be shown that $\frac{D_u}{a+2-2\sqrt{a}} \geq \frac{1+a}{2} D_u$.

We examine the profit functions $\overline{\Pi}_1 = \overline{\Pi}_1^u + \overline{\Pi}_1^p$ and $\overline{\Pi}_2 = \overline{\Pi}_2^u + \overline{\Pi}_2^p$ in each region:

1. [U0P1]: $\overline{\Pi}_1 = I_1^u p_{\max}$ and $\overline{\Pi}_2 = I_2^u p_{\max}$. Firm 2 deviates to U2P1.

2. [U1P0]: $\overline{\Pi}_1 = I_1^p p_{\max}$ and $\overline{\Pi}_2 = I_2^p p_{\max} + D_u\Delta$. Firm 1 deviates to U6P2.
3. [U1P2]: $\overline{\Pi}_1 = (D_p - I_2^p)p_{\max}$ and $\overline{\Pi}_2 = \frac{I_2^p}{D_p}(D_p - I_2^p)p_{\max} + D_u\Delta$. Firm 1 deviates to U6P2.
4. [U1P4]: $\overline{\Pi}_1 = (D_p - I_2^p)p_{\max}$ and $\overline{\Pi}_2 = \frac{I_2^p}{I_1^p}(D_p - I_2^p)p_{\max} + D_u\Delta$. Firm 1 deviates to U6P2.
5. [U1P5]: $\overline{\Pi}_1 = \frac{I_1^p}{I_2^p}(D_p - I_1^p)p_{\max}$ and $\overline{\Pi}_2 = (D_p - I_1^p)p_{\max} + D_u\Delta$. Firm 2 deviates to U4P3.
6. [U2P1]: $\overline{\Pi}_1 = (D_u - I_2^u)p_{\max}$ and $\overline{\Pi}_2 = I_2^u p_{\max}$. Firm 1 deviates to U2P3 or to U4P3.
7. [U2P3]: $\overline{\Pi}_1 = \frac{I_1^p}{D_p}(D_p - I_1^p)p_{\max} + (D_u - I_2^u)p_{\max}$ and $\overline{\Pi}_2 = (D_p - I_1^p)p_{\max} + I_2^u p_{\max}$. Firm 2 deviates to U4P3.
8. [U3P1]: $\overline{\Pi}_1 = (D_u - I_2^u)p_{\max}$ and $\overline{\Pi}_2 = I_2^u\left(\frac{D_u - I_2^u}{I_1^u}p_{\max} + \Delta\right)$. Firm 1 deviates to U4P3.
9. [U4P3]: $\overline{\Pi}_1 = \frac{I_1^p}{D_p}(D_p - I_1^p)p_{\max} + (D_u - I_2^u)p_{\max}$ and $\overline{\Pi}_2 = (D_p - I_1^p)p_{\max} + I_2^u\left(\frac{D_u - I_2^u}{D_u}p_{\max} + \Delta\right)$. By the first-order condition, the only equilibrium candidate is $(I_1^u, I_2^u) = \left(I - \frac{D_p}{2}, \frac{1+a}{2}D_u\right)$. We next check whether there is any profitable deviation:
   - Given $I_1^u = I - \frac{D_p}{2}$, if Firm 2 deviates, the best choice would be $I_2^u = I - \frac{D_p}{2}$ in U1P5, which yields $\frac{D_p}{2}p_{\max} + D_u\Delta \le \frac{D_p}{2}p_{\max} + (\frac{1+a}{2})^2 D_u p_{\max}$. Thus, Firm 2 does not deviate.
   - Given $I_2^u = \frac{1+a}{2}p_{\max}$, Firm 1 has no incentive to deviate.

   Therefore, $(I_1^u, I_2^u) = (I - \frac{D_p}{2}, \frac{1+a}{2}D_u)$ is an equilibrium.
10. [U5P1]: $\overline{\Pi}_1 = I_1^u\left(\frac{D_u - I_1^u}{I_2^u}p_{\max} - \Delta\right)p_{\max}$ and $\overline{\Pi}_2 = (D_u - I_1^u)p_{\max}$. Firm 2 deviates U6P2.
11. [U6P2]: $\overline{\Pi}_1 = (D_p - I_2^p)p_{\max} + I_1^u\left(\frac{D_u - I_1^u}{D_u}p_{\max} - \Delta\right)p_{\max}$ and $\overline{\Pi}_2 = \frac{I_2^p}{D_p}(D_p - I_2^p)p_{\max} + (D_u - I_1^u)p_{\max}$. By the first-order condition, the only equilibrium candidate is $(I_1^u, I_2^u) = \left(\frac{1-a}{2}D_u, I - \frac{D_p}{2}\right)$. We next check whether there is any profitable deviation:
   - Given $I_2^u = I - \frac{D_p}{2}$, Firm 1 has no incentive to deviate.
   - Given $I_1^u = \frac{1-a}{2}D_u$, Firm 2 obviously has no incentive to deviate to U5P1. Along the vertical line $I_1^u = \frac{1-a}{2}D_u$ in U3P1, Firm 2's profit, $I_2\left(\frac{D_i - I_2^u}{I_1^u}p_{\max} + \Delta\right)$, is decreasing in I_2^u; and in U2P1, Firm 2's profit, $I_2^u p_{\max}$, is increasing in I_2^u. Thus, in order for $(I_1^u, I_2^p) = \left(\frac{1-a}{2}D_u, \frac{D_p}{2}\right)$ to be an equilibrium, we need $\frac{D_p}{4}p_{\max} + \frac{1+a}{2}D_u p_{\max} \ge I_2^u p_{\max}$ when $I_2^u = D_u - (1-a)\frac{1-a}{2}D_u$, that is:
   $$\frac{D_p}{4}p_{\max} + \frac{1+a}{2}D_u p_{\max} \ge \left[D_u - \frac{(1-a)^2}{2}D_u\right]p_{\max} \iff \frac{D_p}{D_u} \ge 2a(1-a).$$

   Therefore, $(I_1^u, I_2^u) = \left(\frac{1-a}{2}D_u, I - \frac{D_p}{2}\right)$ is an equilibrium if and only if $\frac{D_p}{D_u} \ge 2a(1-a)$.

In conclusion, $E_A = (I - \frac{D_p}{2}, \frac{1+a}{2}D_u)$, located in U4P3, is always a Nash equilibrium, and $E_B = \left(\frac{1-a}{2}D_u, I - \frac{D_p}{2}\right)$, which is located in U6P2, is a Nash equilibrium if and only if $\frac{D_p}{D_u} \ge 2a(1-a)$. □

REMARK 1. For simplicity we have assumed that $D = I$. When $D < I$, the expressions of the equilibria remain identical and the equilibrium profits do not change.

Appendix C: Pooled Capacity vs. Pro Rata Capacity Allocation

In the main paper, we studied uniform pricing with pooled capacity, discriminatory pricing with pro rata capacity allocation, and discriminatory pricing with endogenous inventory allocation. In this appendix, we discuss two other strategies: discriminatory pricing with pooled capacity and uniform pricing with pro rata capacity allocation. In fact, these cases are either identical (under low demand) or generate similar insights (under high demand) as those studied in the main paper.

C.1. Low-Demand Regime: Equivalence of Pooled Capacity and Pro Rata Allocation

First, uniform pricing with pro rata capacity allocation is equivalent to uniform pricing with pooled capacity. Under pro rata allocation, both firms allocate their capacities to each segment proportionally to its size. Therefore, the inventory allocated by each firm to QSC is $I_1^u = I_2^u = \frac{D_u}{D} I \geq D_u$, whereas the inventory allocated by each firm to PSC is given by $I_1^p = I_2^p = \frac{D_p}{D} I \geq D_p$. This implies that no firm faces a capacity shortage in any of the two segments—exactly as in the pooled capacity case and unlike the endogenous allocation case. The price competition game is thus identical to that under pooled capacity (Section 4.1).

Next, discriminatory pricing with pooled capacity is equivalent to discriminatory pricing with pro rata capacity allocation. Under discriminatory pricing with pooled capacity, both firms have a total inventory I across both segments and compete only on price. Let p_i^p and p_i^u denote the price charged by Firm i to PSC and QSC, respectively. The demand faced by each firm is identical to the demand under pooled capacity. Again, under low demand, the inventory does not come into play, and the price competition is thus independent across the two segments. Stated differently, the pricing decisions can be made independently on the PSC segment and on the QSC segment, which is again equivalent to the case with pro rata capacity allocation.

C.2. High-Demand Regime

Under high demand, the game with discriminatory pricing and pooled capacity is underspecified under the assumptions imposed in our paper. For instance, if $p_1^p < p_2^p$ and $p_1^u + \Delta < p_2^u$, Firm 1's faces a demand of D but a capacity of $I < D$. We would thus additional rationing rules across segments; in our example, Firm 1 can use up to $\min\{D_p, I\}$ for PSC and the rest for QSC, or $\min\{D_u, I\}$ for QSC and the rest for PSC.

The case of uniform pricing with pro rata capacity allocation is defined as follows:

$$I_1^p = I_2^p = \frac{I}{D} D_p \in [0.5D_p, 0.625D_p] \quad \text{and} \quad I_1^u = I_2^u = \frac{I}{D} D_u \in [0.5D_u, 0.625D_u].$$

The demand structure is given by:

$$D_1(p_1,p_2) = \begin{cases} I & \text{if } p_1+\Delta < p_2 \\ \frac{D_u}{2} + I_1^p & \text{if } p_1+\Delta = p_2 \\ D_u - I_2^u + I_1^p & \text{if } p_1 < p_2 < p_1+\Delta \\ D_u - I_2^u + \frac{D_p}{2} & \text{if } p_1 = p_2 \\ D - I & \text{if } p_1 > p_2 \end{cases} = \begin{cases} I & \text{if } p_1+\Delta < p_2 \\ I - (\frac{I}{D} - \frac{1}{2})D_u & \text{if } p_1+\Delta = p_2 \\ I - (\frac{2I}{D} - 1)D_u & \text{if } p_1 < p_2 < p_1+\Delta \\ \frac{D}{2} - (\frac{I}{D} - \frac{1}{2})D_u & \text{if } p_1 = p_2 \\ D - I & \text{if } p_1 > p_2 \end{cases}$$

and $D_2(p_2,p_1) = D - D_1(p_1,p_2)$. The profit functions are given by:

$$\begin{aligned} \pi_1(p_1, F_2) &= p_1 \Big\{ I - \left(\tfrac{2I}{D} - 1\right) D_u F_2(p_1+\Delta) + \left(\tfrac{I}{D} - \tfrac{1}{2}\right) D_u Q_{F_2}(p_1+\Delta) \\ &\quad - \left(\tfrac{2I}{D} - 1\right) D_p F_2(p_1) + \left(\tfrac{I}{D} - \tfrac{1}{2}\right) D_p Q_{F_2}(p_1) \Big\} \\ \pi_2(p_2, F_1) &= p_2 \Big\{ I - \left(\tfrac{2I}{D} - 1\right) D_u F_1(p_2-\Delta) + \left(\tfrac{I}{D} - \tfrac{1}{2}\right) D_u Q_{F_1}(p_2-\Delta) \\ &\quad - \left(\tfrac{2I}{D} - 1\right) D_p F_1(p_2) + \left(\tfrac{I}{D} - \tfrac{1}{2}\right) D_p Q_{F_2}(p_1) \Big\}. \end{aligned}$$

Proposition 16. *The following mixed-strategies form a Nash equilibrium under uniform pricing with pro rata capacity allocation (the proof mirrors that of Proposition 10):*

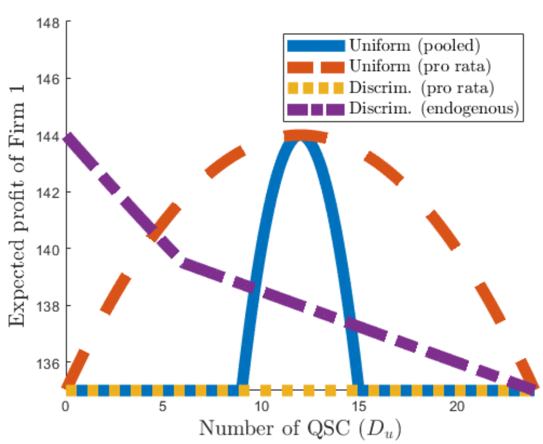
$$F_1(p) = \frac{I - \frac{\overline{\Pi}_2^{UF-PR}}{p}}{(\frac{2I}{D} - 1)D_p}, \ p \in [\underline{p}, p_{max}],$$

$$F_2(p) = \frac{I - (\frac{2I}{D} - 1)D_u - \frac{\overline{\Pi}_1^{UF-PR}}{p}}{(\frac{2I}{D} - 1)D_p}, \; p \in [\underline{p}, p_{max}),$$
$$Q_{F_2}(p_{max}) = 1 - \lim_{p \to p_{max}+} F_2(p),$$

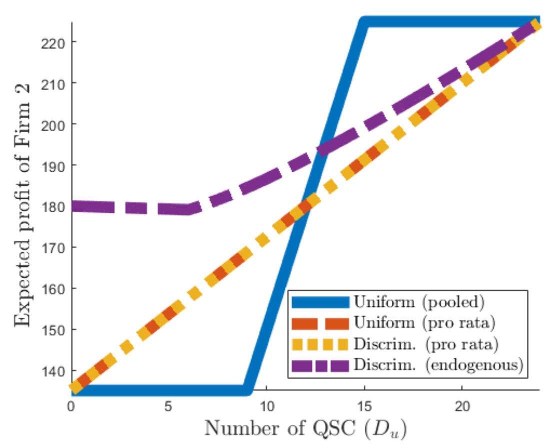
where $\overline{\Pi}_1^{UF-PR} = [I - (\frac{2I}{D} - 1)D_u]\underline{p} = \frac{[D-I+(\frac{2I}{D}-1)D_u][I-(\frac{2I}{D}-1)D_u]}{I} p_{max}$ *and* $\overline{\Pi}_2^{UF-PR} = [D - I + (\frac{2I}{D} - 1)D_u]p_{max}$ *are the expected profit and* $\underline{p} = \frac{\overline{\Pi}_2^{UF-PR}}{I}$.

Comparing the profit expressions (shown in Figure 12) yield the following insights:

- Under uniform pricing (with pooled capacity or pro rata capacity allocation), Firm 1 benefits from a more heterogeneous market, whereas Firm 2 benefits from a larger QSC customer pool. This extends our result on the market frictions introduced by quality differentiation and customer heterogeneity.
- Under pro rata capacity allocation, Firm 2 is indifferent between uniform and discriminatory pricing, whereas Firm 1 is strictly better off under uniform pricing. This further emphasizes the market friction brought by customer heterogeneity, which benefits the firm with the lower-quality product. Discriminatory pricing removes the customer heterogeneity friction and, under pro rata capacity allocation, does not introduce extra friction from capacity restrictions—which results in lower profits for Firm 1.
- Uniform pricing (with pooled capacity or pro rata capacity allocation) can benefit Firm 1 relative to price discrimination (with pro rata and even endogenous allocation). This confirms the joint roles of market frictions from quality differentiation, customer heterogeneity, and capacity restrictions.



(a) Firm 1



(b) Firm 2

Figure 12 **Expected profits as functions of D_u (parameters: $D = 24$, $I = 15$, $\Delta = 6$, and $p_{\max} = 15$).**

Appendix D: Heterogeneous Market: Moderate-Demand Regime

We study the case where $I < D < \left(2 - \frac{\Delta}{p_{\max}}\right) I$. As we shall see, our insights derived under low and high demand hold will be robust to customer demand and market capacity.

D.1. Complexity of the Problem Under Moderate Demand

The characterization of the Nash equilibria becomes much more intricate under moderate demand than under low and high demand—both under uniform pricing and under discriminatory pricing. Under uniform pricing, Firm 1's profit when playing p_1 depends on Firm 2's mixed strategy both at p_1 and at $p_1+\Delta$; similarly, Firm 2's profit when playing p_2 depends on Firm 1's mixed strategy both at p_2 and at $p_2-\Delta$:

$$\pi_1(p_1,p_2)=\begin{cases} p_1\times I & \text{if } p_1+\Delta<p_2\\ p_1\times\min\left(\frac{D_u}{2}+D_p,I\right) & \text{if } p_1+\Delta=p_2\\ p_1\times\min\left(D_p+\max(D_u-I,0),I\right) & \text{if } p_2<p_1+\Delta<p_2+\Delta\\ p_1\times\min\left(\frac{D_p}{2}+\max\left(D_u+\frac{D_p}{2}-I,0\right),I\right) & \text{if } p_2<p_1+\Delta<p_2+\Delta\\ p_1\times(D-I) & \text{if } p_2<p_1+\Delta<p_2+\Delta \end{cases}$$

$$\pi_2(p_1,p_2)=\begin{cases} p_2\times(D-I) & \text{if } p_1+\Delta<p_2\\ p_2\times\min\left(\frac{D_u}{2}+\max\left(D_p+\frac{D_u}{2}-I,0\right),I\right) & \text{if } p_1+\Delta=p_2\\ p_2\times\min\left(D_u+\max(D_p-I,0),I\right) & \text{if } p_2<p_1+\Delta<p_2+\Delta\\ p_1\times\min\left(\frac{D_p}{2}+D_u,I\right) & \text{if } p_2<p_1+\Delta<p_2+\Delta\\ p_2\times I & \text{if } p_2<p_1+\Delta<p_2+\Delta. \end{cases}$$

This structure induces interdependencies throughout the interval $[0,p_{\max}]$. Unlike under low and high demand, the structure of the equilibrium varies drastically within sub-regimes, defined as a function of I, $p_{\max}$, D_u, and D_p. For example, there exist cases where the support of each firm's mixed strategy is the union of several non-overlapping intervals. The exhaustive characterization of Nash equilibria over the entire parameter space is analytically intractable under moderate demand. Similarly, under discriminatory pricing with endogenous inventory allocation, the number of different combinations grows significantly as compared to the already-complex high-demand regime. We therefore characterize Nash equilibria computationally.

D.2. Uniform Pricing

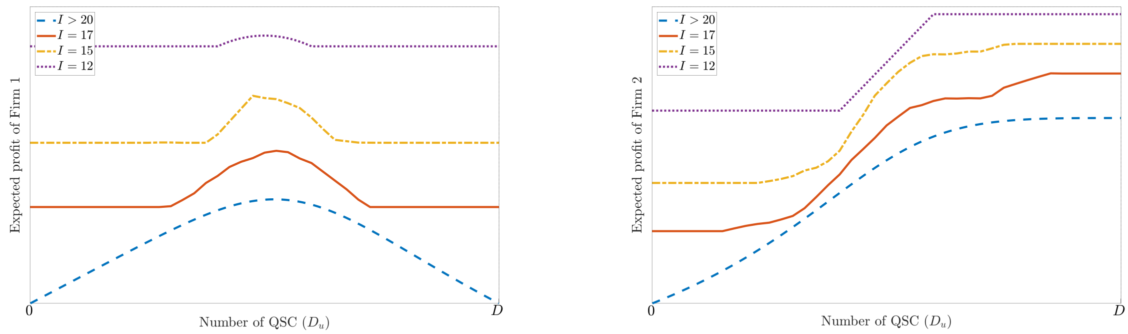
We discretize $[0,p_{\max}]$ into a finite set $p_k=\frac{k}{N}p_{\max}$, for $k=0,...,N$. The resulting game has a finite strategy space, and thus admits a (mixed-strategy) Nash equilibrium. We denote by $\boldsymbol{x}$ and $\boldsymbol{y}$ the probability vectors characterizing Firm 1's and Firm 2's mixed strategies, and define two $(N+1)\times(N+1)$ matrices $\boldsymbol{A}$ and $\boldsymbol{B}$, where $A_{kl}=\pi_1(p_k,p_l)$ and $B_{kl}=\pi_2(p_k,p_l)$ for all $k,l=0,\cdots,N$. Then, $(\boldsymbol{x}^*,\boldsymbol{y}^*)$ is a mixed-strategy Nash equilibrium if and only if there exist λ^*,μ^* such that $(\boldsymbol{x}^*,\boldsymbol{y}^*,\lambda^*,\mu^*)$ solves the following bilinear program:

$$\begin{aligned} \max\quad & \boldsymbol{x}^\top\boldsymbol{A}\boldsymbol{y}+\boldsymbol{x}^\top\boldsymbol{B}\boldsymbol{y}-\lambda-\mu\\ \text{s.t.}\quad & \boldsymbol{A}\boldsymbol{y}\le\lambda\boldsymbol{1},\quad \boldsymbol{B}^\top\boldsymbol{x}\le\mu\boldsymbol{1},\\ & \boldsymbol{x}^\top\boldsymbol{1}=1,\quad \boldsymbol{y}^\top\boldsymbol{1}=1,\\ & \boldsymbol{x}\ge 0,\quad \boldsymbol{y}\ge 0, \end{aligned}$$

where $\boldsymbol{1}$ denotes the $(N+1)$-dimensional vector with all components equal to 1. At the equilibrium, the expected profits of Firm 1 and Firm 2 are $\lambda^*=(\boldsymbol{x}^*)^\top\boldsymbol{A}\boldsymbol{y}^*$ and $\mu^*=(\boldsymbol{x}^*)^\top\boldsymbol{B}\boldsymbol{y}^*$, respectively.

We could solve all instances of the above bilinear optimization problem in reasonable timeframes with $N=400$. We validated our discretization approach using low-demand and high-demand instances of the problem, to verify that the solution coincides with the analytical Nash equilibria.

Figure 13 depicts both firms' expected profits as functions of D_u when $D=20$, $p_{\max}=13$, and $\Delta=5$, for different values of I. When $I>20$, the game falls into the low-demand regime; when $I=12$, the game falls into high demand; in-between, the game falls into moderate demand.



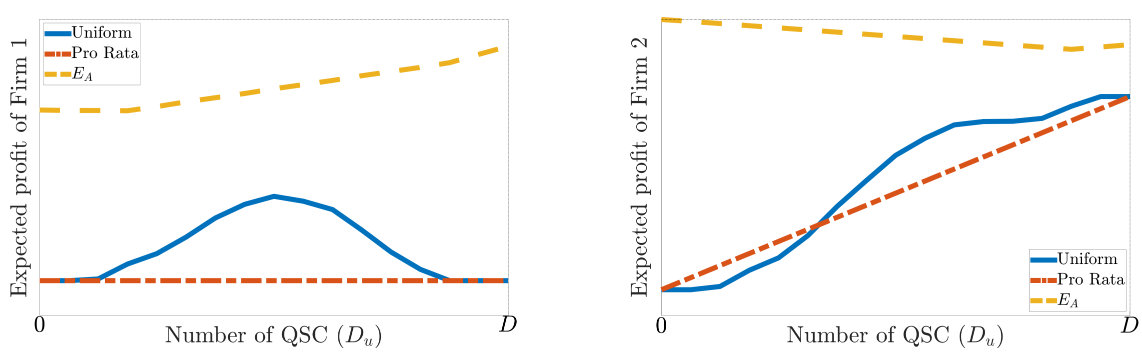
(a) Firm 1 (b) Firm 2

Figure 13 Expected profits as functions of D_u (parameters: $D = 20$, $p_{\max} = 13$, and $\Delta = 5$).

Note, first, that for a given D_u, each firm's profit decreases with I: both firms are better off as the game becomes more capacitated, which induces higher market frictions. Second, and most importantly, Firm 1's expected profit is highest when D_u is close to (or equal to) $\frac{D}{2}$, whereas Firm 2's expected profit is increasing with $D_u \in [0, D]$, confirming that Firm 2 benefits from quality sensitivity whereas Firm 1 benefits from customer heterogeneity. Third, the impact of customer heterogeneity is more significant for high values of I. Indeed, for small values of I, the market friction from capacity restrictions dominates the friction from customer heterogeneity; vice versa, large values of I induce stronger frictions from customer heterogeneity.

D.3. Discriminatory Pricing

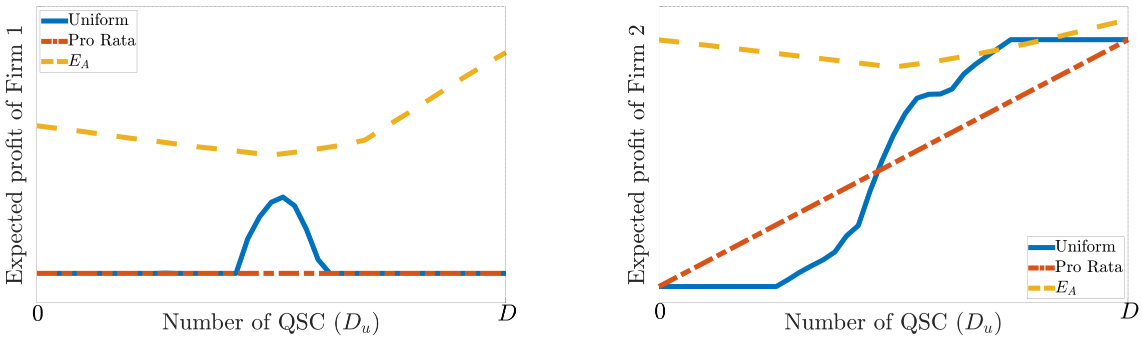
We now turn to price discrimination with endogenous inventory allocation. Again, we discretize each firm's strategy space, compute expected profits using Table 1, and determine the Nash equilibrium computationally. Multiple equilibria exist in general, but one exists consistently across all parameter values—one where Firm 1 allocates most of its inventory to QSC and Firm 2 allocates most of its inventory to PSC. We still refer to it as E_A. Figures 14–16 depict both firms' expected profits as functions of D_u, under each strategy.



(a) Firm 1 (b) Firm 2

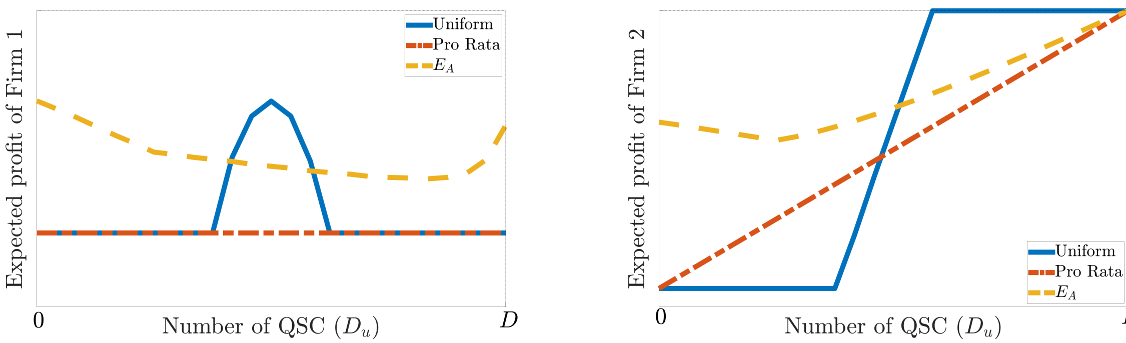
Figure 14 Expected profits as functions of D_u (parameters: $D = 16$, $I = 15$, $p_{\max} = 15$, and $\Delta = 5$).

Once again, most insights established under low and high demand still hold in the moderate-demand regime, thus, establishing the robustness of our findings. Specifically, we observe the following:



(a) Firm 1 (b) Firm 2

Figure 15 Expected profits as functions of D_u (parameters: $D=20$, $I=15$, $p_{\max}=15$, and $\Delta=5$).



(a) Firm 1 (b) Firm 2

Figure 16 Expected profits as functions of D_u (parameters: $D=24$, $I=15$, $p_{\max}=15$, and $\Delta=5$).

- When $D=16$, which is slightly higher than the inventory $I=15$, Figure 14 exhibits a similar pattern to the low-demand regime (Figure 5), both in terms of profit variations and profit comparisons. Our insights from the low-demand regime hold: price discrimination is not necessarily beneficial under competition, but firms can increase their profits by combining discriminatory pricing with strategic inventory allocation to strengthen market frictions. One difference with the low-demand case is that uniform pricing no longer dominates discriminatory pricing with pro rata capacity allocation across the entire space, an indication of the transition from low demand to high demand.
- When $D=20$, Figure 15 shows that discriminatory pricing with endogenous capacity allocation remains overall beneficial, but its benefits become smaller as demand increases.
- When $D=24$, which is slightly lower than $\left(2-\frac{\Delta}{p_{\max}}\right)I=25$, Figure 16 exhibits a similar pattern to the high-demand regime (Figure 9). Again, our insights are preserved: (i) under price discrimination, endogenous inventory allocation outperforms pro rata allocation, and (ii) there is no strict dominance between the uniform and discriminatory pricing strategies. In fact, we now observe an additional region where uniform pricing may outperform discriminatory pricing (even with endogenous capacity allocation) in the face of customer heterogeneity, which expands our insights derived under high demand.

Appendix E: Sensitivity to α

Recall that we define the quality differential as $\Delta = \alpha(\mu_2 - \mu_1)$, so the parameter α governs the extent of quality differentiation between Firm 1's and Firm 2's products. In this appendix, we conduct a sensitivity analysis with respect to α, reported in Table 3.

Table 3 Sensitivity of firms' profits as a function of the quality-sensitivity parameter α.

Demand	Pricing	Inventory	Firm 1 profit	Firm 2 profit	Total profit
Low	Uniform	—	increases with α	increases with α	increases with α
	Discriminatory	Pro rata	indep. of α (zero)	increases with α	increases with α
	Discriminatory	Endogenous	decreases with α	increases with α	increases with α
High	Uniform	—	independent of α	independent of α	independent of α
	Discriminatory	Pro rata	independent of α	independent of α	independent of α
	Discriminatory	Endogenous	decreases with α	increases with α	increases with α

As $\alpha \to 0$, the game converges to market homogeneity. It features (i) a low-demand regime where $D \leq I$; (ii) a moderate-demand regime where $I < D < 2I$; and (iii) a high-demand regime where $D = 2I$. A shrinking quality differential makes the high-demand regime disappear, precisely because this regime was characterized by the fact that both firms were able to leverage high demand and quality differences to charge high prices. Under low demand, the game is characterized by the well-known price war, in which both firms earn zero profit under uniform pricing or discriminatory pricing with pro rata inventory allocation. However, under discriminatory pricing with endogenous inventory allocation, the firms can still create inventory shortages through strategic rationing, thus creating market frictions and earning positive profits.

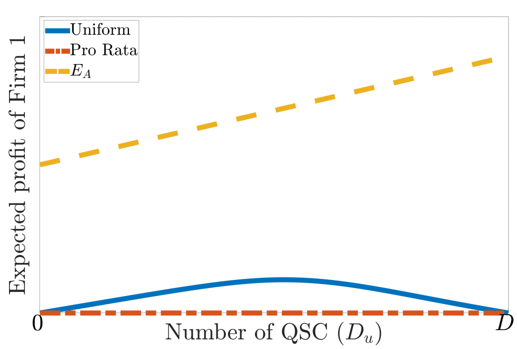
As α increases, the quality sensitivity of the relevant segment becomes stronger—namely, quality-sensitive customers become "more quality sensitive." One could expect that Firm 2 will benefit more from its quality advantage. A second question is whether Firm 1's profits will increase or decrease. The answer depends on market demand and, most importantly, on the firms' pricing strategies (see Figures 17–20 below):

- Under low demand and uniform pricing, both firms' profits increase with α. Firm 2 increases its profits by attracting quality-sensitive customers more easily. One could then expect that, conversely, Firm 1 will be hurt by stronger quality sensitivity. However, this is not the case. Again, this stems from the fact that a higher value of α makes it harder for Firm 2 to serve both customer segments simultaneously: stronger quality sensitivity creates incentives for Firm 2 to charge higher prices but higher prices also hurt Firm 2's ability to attract price-sensitive customers. Therefore, Firm 1 can exploit market heterogeneity and increase its own profits. We can thus interpret a higher value of α as stronger quality asymmetry—benefitting Firm 2—or as stronger customer heterogeneity—benefitting Firm 1.
- Under high demand and uniform pricing, both firms' profits are independent of α. In that case, capacity restrictions form the main market friction, and both firms play prices in $[p_{\max} - \Delta, p_{\max}]$. Firm 2 is still able to exploit its quality differential and attract all quality-sensitive customers, thus earning a higher profit than Firm 1. However, the firms' strategies and profits are unaffected by α. Still, higher values of α increase the incidence of the high-demand regime, characterized by $D \geq \left(2 - \frac{\Delta}{p_{\max}}\right) p_{\max}$, increasing the firms' abilities to extract surplus from quality differentiation and inventory restrictions.

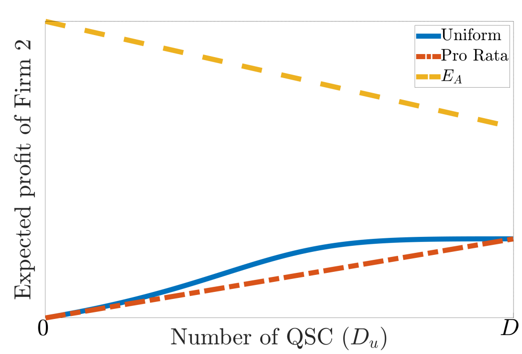
– Under discriminatory pricing with pro rata inventory allocation, Firm 1's profit is independent of α. Indeed, this pricing strategy eliminates the market frictions from quality differentiation and customer heterogeneity. Under low demand, it also eliminates market frictions from capacity restrictions, so Firm 1 earns zero profit. Firm 2 still earns a positive profit by charging Δ, exploiting its quality advantage to attract all quality-sensitive customers. Thus, Firm 2's profit increases with α. Under high demand, the competition is primarily driven by the market friction induced by capacity restrictions, as opposed to quality differentiation, and both firms' profits are unaffected by α.

– Under discriminatory pricing with endogenous inventory allocation, Firm 2's profit increases with α and Firm 1's profit decreases with α. Again, price discrimination eliminates the market frictions induced by quality differentiation and customer heterogeneity. Since this friction was the driving force behind the positive impact of α on Firm 1's profits under uniform pricing, a higher value of α is now detrimental to Firm 1. In contrast, Firm 2's profit is still increasing in α due to its quality advantage.

Let us detail these dynamics in Equilibrium E_A. Under low demand, Firm 1 creates an inventory shortage in the PSC segment and Firm 2 creates a shortage in the QSC segment for each firm to "focus" on the segment where it has a more natural advantage. Price competition then unfolds independently in the two segments. In the PSC segment, quality advantage does not play a role, so both firms' profits are independent of α. In the QSC segment, Firm 2 benefits from its higher-quality product—by allocating more capacity to the QSC segment and charging higher prices. As a result, Firm 2's profit increases in α, whereas Firm 1's profit decreases in α. The dynamics are similar under high demand except that the higher capacity allocated by Firm 2 in the QSC segment creates a stronger shortage in the PSC segment. As a result, Firm 1's profit from PSC benefits from a higher α; nonetheless, a higher quality differential still leads to lower profits for Firm 1 in the QSC segment, and lower overall profits.



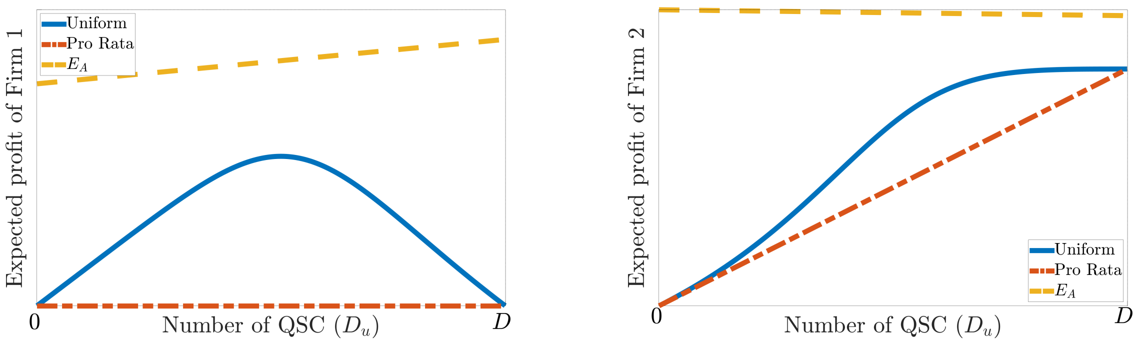
(a) Firm 1



(b) Firm 2

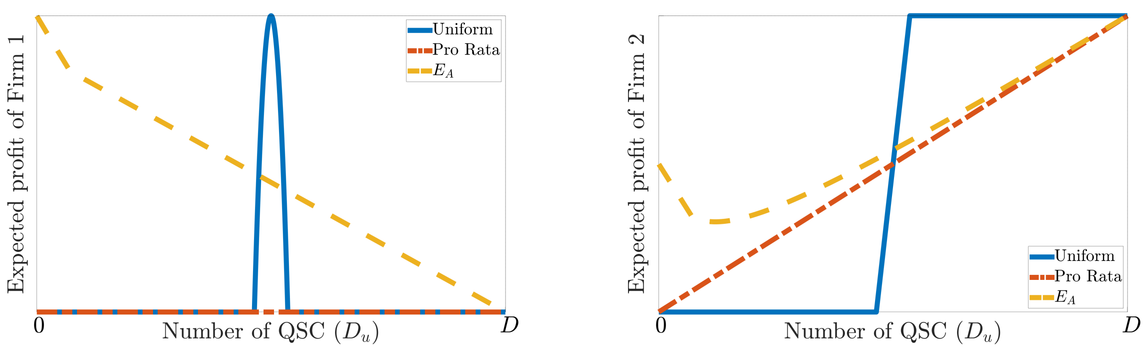
Figure 17 Expected profits as functions of D_u (parameters: $D = 12$, $I = 15$, $p_{\max} = 15$, and $\Delta = 2$).

In summary, the sensitivity analysis with respect to the quality differential α yields two takeaways. First, the quality differential has a different effect under uniform pricing vs. discriminatory pricing. Indeed, under uniform pricing, a stronger quality differential benefits both firms: the firm with the higher-product quality benefits from a more quality-sensitive customer pool, whereas the firm with the lower-quality product benefits



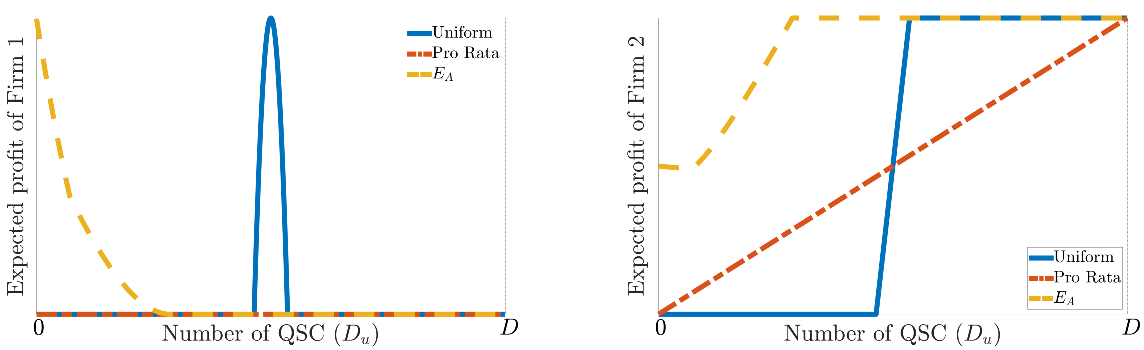
(a) Firm 1 (b) Firm 2

Figure 18 **Expected profits as functions of D_u (parameters: $D = 12$, $I = 15$, $p_{\max} = 15$, and $\Delta = 6$).**



(a) Firm 1 (b) Firm 2

Figure 19 **Expected profits as functions of D_u (parameters: $D = 28$, $I = 15$, $p_{\max} = 15$, and $\Delta = 2$).**



(a) Firm 1 (b) Firm 2

Figure 20 **Expected profits as functions of D_u (parameters: $D = 28$, $I = 15$, $p_{\max} = 15$, and $\Delta = 6$).**

from a more heterogeneous customer pool. In contrast, under discriminatory pricing, a stronger quality differential benefits the firm with the higher-product quality but hurts the firm with the lower-product quality. Second, the quality differential has a different effect under discriminatory pricing with pro rata vs. endogenous capacity allocation. Specifically, pro rata allocation alleviates the market frictions from capacity restrictions, quality differentiation, and customer heterogeneity. Under endogenous capacity allocation, the firm with the higher-quality product can create capacity shortages to exploit its advantage.