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Consumer Subsidies with a Strategic Supplier: Commitment vs. Flexibility

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Abstract. Governments use consumer incentives to promote green technologies (e.g., solar panels and electric vehicles). Our goal in this paper is to study how policy adjustments over time will interact with production decisions from the industry. We model the interaction between a government and an industry player in a two-period game setting under uncertain demand. We show how the timing of decisions affects the risk sharing between the government and the supplier, ultimately affecting the cost of the subsidy program. In particular, we show that when the government commits to a fixed policy, it encourages the supplier to produce more at the beginning of the horizon. Consequently, a flexible subsidy policy is on average more expensive, unless there is a significant negative demand correlation across time periods. However, we show that the variance of the total sales is lower in the flexible setting, implying that the government's additional spending reduces the adoption level uncertainty. In addition, we show that for flexible policies, the supplier is better off in terms of expected profits, whereas the consumers can either benefit or not depending on the price elasticity of demand. Finally, we test our insights with a numerical example calibrated on data from a solar subsidy program.

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1. Introduction

To stimulate the adoption of a new technology, governments have typically introduced policy interventions to subsidize customers. Examples of such subsidy programs in Europe and the United States are common in the renewable energy sector, where feed-in-tariffs and rebates have helped promoting solar and wind technologies. In Germany, solar electricity contributed to roughly 4.6% of the total electricity consumption in 2012. Combined with wind power expansion, the country is well on its way to reaching the long-term goal of 35% renewable energy by 2020. Dating back to 2001, the German feed-in-tariff program initially paid solar panel owners 0.5062 euros per kWh of electricity produced, more than three times the average retail electricity price. This feed-in-tariff system kick-started a new solar industry, and by the end of 2011, there were more than 24.7 GW of installed photovoltaic capacity in Germany, which represents roughly 37% of the total installed capacity worldwide. Over the years 2010–2012, Germany has added a consistent 7.4–7.6 GW per year of photovoltaic capacity. Achieving this target has not been an easy feat. In 2012, the German feed-in-tariff level changed four times throughout the year.[1] The feed-in-tariff program entered a new phase in 2012, as it tried to control the rate of adoption. The current policy follows a monthly digression rate that depends on previously installed capacity of photovoltaics. In 2013 and 2014, the government hoped to achieve a yearly installation target of 2.5–3.5 GW of new PV installations by continuing to adjust the subsidy level multiple times a year.[2] The effects of these policy adjustments on the solar industry is not clear yet, and this research aims to shed new light on this question.

The California Solar Initiative (CSI) is another example of solar subsidy policy. This program hoped to install 1,940 MW of solar panels between 2007 and 2016.[3] As part of the original design of the program, there was a planned decrease of the incentive amount. The different subsidy levels used were preannounced in a way that would phase out the program. Over the last 10 years, the CSI subsidies have followed the planned phase out, which can be interpreted as a policy commitment. Hughes and Podolefsky (2015) provide a good overview of the program. In particular, the authors discuss how solar installation firms reacted in anticipation to the predictable changes in the subsidy level.

The federal tax credit for plug-in electric vehicles is an additional example of subsidy program. Introduced in 2009, the U.S. government provides a consumer subsidy of $\$7,500$ for the purchase of an electric vehicle.[4] Unlike the solar subsidy mentioned above, this subsidy amount has not changed since its inception. In this paper, our goal is to understand the benefits and disadvantages of such a policy commitment. Additionally, we study the implications of the timing of government decisions for the industry and the consumers.

Governments often try to commit to environmental policies for several years. However, in many cases, governments renege on their commitments. The question of whether governments have the ability to commit to long-term policy decisions is an active subject of debate. While we do not attempt to prove that governments can commit in practice, we intend to study what would happen to subsidy programs if they could commit. In the CSI and EV policies mentioned above, the subsidy programs did follow the preannounced path. In other examples, however, the commitment was not enforced because of legal and political obstacles.

In this paper, we impose the modeling assumption that under the committed setting, the government cannot revise its subsidy policy. In reality, new governments are regularly elected and may decide to continue or discontinue existing policies. For example, in August 2015, a law was passed in the Israeli parliament stating that the government is not allowed to revise any details of a natural gas regulation within the next 10 years.[5] Nevertheless, this law was struck down by the Supreme Court a few months after the vote. Note that this is an example where the government was trying to commit but ultimately failed. Consequently, such commitments cannot always be achieved. This paper aims to understand if such commitments are valuable in practice, and whether one should strive to achieve it.

Three recent examples are observed from the French government, where commitments for green technology investments were successfully implemented. The first example relates to the electric vehicles industry and fits very well the setting studied in this paper. Since 2014, the French government has been rewarding any resident who purchases an electric vehicle with a financial "bonus." This bonus amounts to 27% of the cost (with a limit of 6,300€). The French government committed to this amount up front for the years 2015 and 2016. This ministerial ruling is a perfect illustration of a government committing to a rebate policy. The official page (in French) of the law can be found at https://www.service-public.fr/particuliers/vosdroits/F32430. Another interesting feature of this law is the incentive for buying nonelectric but low-emission vehicles (between 21 and 60 g/km). For any vehicle ordered before January 2016, the bonus amounted to 20% of the cost of the car (with a limit of 4,000€). After April 2016, the subsidy amounts to 1,000€. This is an illustration of a commitment to a rebate policy with different subsidy levels in each period. The official page (in French) of the part describing this regulation can be found at https://www.service-public.fr/particuliers/vosdroits/F18132. A third example from the French government can be found in green housing renovations. In particular, a resident who undertakes specific home renovations (with the goal of making the house more eco-friendly) and spends a minimum of 15,000€ will benefit from a five-year property-tax exemption (50% or 100% depending on the decision of the local council). Here, again, the government commits up front to a five-year horizon. The official page (in French) of the French government describing the property tax and discussing the exemptions can be found at https://www.service-public.fr/particuliers/vosdroits/F59.

The value of commitment in public policies has also been studied in other contexts. As summarized by Dixit and Pindyck (1994, p. 50), "if an objective of public policy is to stimulate investment, the stability of interest rates may be more important than the level of interest rates." This insight is derived by Ingersoll and Ross (1992), who show that the interest rate uncertainty delays investments. On the other hand, one can find situations where uncertainty is not as harmful to investments. For example, Kulatilaka and Perotti (1998) show that interest rate volatility actually increases the incentive for early investment under a competitive environment. In this paper, we explore this question of policy commitment in the area of subsidy policies. In particular, we measure the trade-off between commitment and flexibility with respect to the production incentives of the suppliers.

In fact, we study how policy revisions interact with a strategic supplier in this market. The anticipation of a policy change decreases the supplier's production target and may increase the overall cost of the subsidy program. Should the government commit their subsidy levels for a longer period of time, or should the subsidy policy be adapted to the realized market demand after each period?

To answer these questions, we model the system as a two-period game between the government and the supplier. The government chooses the subsidy levels for each period, and then the supplier chooses its production levels. We focus on a lost sales model but also consider the case with backorders in our extensions section. Demand is uncertain, so the supplier solves a multiperiod newsvendor problem. We compare two game settings: the government commits to a fixed subsidy policy for each period in advance; or the government has a flexible policy that adapts after the first-period demand is realized.

1.1. Contributions

Under a flexible setting, by holding the option of adjusting the subsidy, the government decreases the underage risk of the supplier. Consequently, this lowers the supplier's initial production level, which we call the undersupply incentive. For this reason, the subsidy levels are on average higher without policy commitment. This effect grows with the magnitude of demand uncertainty, which presents a counterintuitive insight. Instead of a hedging effect, the government spending is more exposed to the variance of the demand uncertainty under a flexible policy.

As a result, when looking at the total average spending, we observe that under a flexible setting, the government typically has to pay a higher cost for achieving the same target adoption level. This difference becomes even larger as demand volatility increases or if the profit margins are high in the second period relative to the first. This result holds even without a strategic supplier or in the presence of competing suppliers. The average flexible spending only becomes lower when there is a strong negative correlation between demands in the two periods. On the other hand, the premium paid for adaptability in the flexible setting provides a lower variance of the sales. We show that the difference in expected spending between the committed and flexible policies is derived by different effects: supply incentive (the supplier produces smaller quantities in the flexible policy), adaptability (the government can adjust the rebates in the second period), and correlation (intertemporal demand correlation). We show that the result on the average government spending holds for both the lost sales and backorders models. However, the supply incentive effect totally disappears for the setting with backorders; hence, we mainly focus on the lost sales model.

Note that if/when there is no law governing the binding of the commitment, the committed and flexible policies are basically the same. In particular, the government would renege at the second time period and revise/adapt its policy. Another way to put it is that the efficacy of the committed policy (that allows for reducing the expected government spending, as we show in this paper) critically depends on the credibility of the government to follow its commitment.

In addition, we compare computationally the total expected welfare between the committed and flexible policies. We convey that under a quadratic externality function, the expected welfare is higher under the flexible setting. As a result, depending on the relative importance of the expected spending (i.e., the budget), the expected total welfare, and the variance of the sales (i.e., the likelihood of reaching a target adoption), the government may decide to adopt the committed or the flexible policy. Consequently, governments with the lack of ability to commit will suffer a higher expected spending but may attain a higher expected welfare.

Firms will always earn higher profits with a flexible government policy. Consumers might prefer the flexible or the committed setting depending on the price sensitivity of demand. In particular, if the price sensitivity in the second period is much higher relative to the first period, the benefits of higher subsidies in the flexible policy are outweighed by the probability of undersupplying high-valued customers.

1.2. Literature Review

There is a growing literature in operations management that studies the impact of subsidy programs. Some develop a prescriptive model for policy optimization—for example, Lobel and Perakis (2017). Alizamir et al. (2016) show that subsidies should not be designed to keep investor profitability constant. Krass et al. (2013) explore the use of environmental taxes to stimulate adoption of green technologies and argue that subsidies should be used to complement the taxes and reduce the welfare loss. Similarly, Terwiesch and Xu (2012) also show that subsidies are often better to stimulate innovation in green technologies than taxes for the polluting technology. Mamani et al. (2012) and Chick et al. (2014) study how to coordinate a vaccines market with subsidies and how to mitigate information asymmetry. It is important to note that the papers above do not explicitly consider demand uncertainty and the resulting mismatch between demand and supply. Kök et al. (2018) model the supply uncertainty from different renewable generation technologies and show how subsidy policies can obtain different outcomes depending on this uncertainty. On the other hand, demand uncertainty can be a significant issue when promoting a green technology product. For example, Sallee (2011) shows that there was a shortage of vehicles manufactured to meet demand when the Toyota Prius was launched. Ho et al. (2002) also show that because of diffusion effects, the firm might want to delay the product launch to build up inventory and avoid a later stockout. This provides further motivation for studying the supplier with a newsvendor model.

Modeling demand uncertainty, Taylor and Xiao (2014) develop a model for how donors should fund malaria drugs through private retailers. They show that donor funding should subsidize purchases not sales of drugs. Raz and Ovchinnikov (2015) compare subsidizing the manufacturer cost and/or consumer purchases in the presence of a single-period price-setting newsvendor. They show that only a joint mechanism can completely coordinate the supply chain, but using only a consumer rebate typically has a small welfare loss. Taylor and Xiao (2017) compare subsidizing of commercial and noncommercial channels. They show that the optimal level of subsidy has a nontrivial relationship with the level of consumer awareness for the product.

Perhaps closer to this paper, Cohen et al. (2016) study the direct impact of demand uncertainty in a single-period game setting between the government and the supplier. They model the supplier as a price-setting newsvendor and show that risk is shared between the supplier and the government depending on the profitability of the product. In contrast, this paper explores a two-period setting and the impact of game dynamics in the risk-sharing between the government and supplier.

Kaya and Özer (2012) provide a good survey of the literature on inventory risk sharing in a supply chain with a newsvendor retailer. Lutze and Özer (2008) show how demand information and inventory risk can be optimally shared in a supply chain with lead times. Babich (2010) shows how a manufacturer can use ordering and subsidy decisions to mitigate the disruption risk from a risky supplier.

The trade-off between commitment and flexibility has been studied in other applications within the operations management literature. In a supply chain context, Erhun et al. (2008) show that the supplier, buyer, and consumers benefit from a multistage dynamic procurement, rather than a single wholesale price contract. Granot and Yin (2007) study how a sequential commitment with buy-back contracts can increase the supplier's profit but harm the retailer. When introducing a new product, Liu and Özer (2010) show that sharing updated demand information to the upstream supplier can provide channel benefits, but a quantity flexibility contract is less robust than a buy-back contract. Kim and Netessine (2013) show that commitment to profit margins can be valuable. It fosters collaboration between supplier and manufacturer, while simple commitments to price or quantity do not. Olsen and Parker (2014) show that inventory commitment can be valuable in a dynamic competition between suppliers.

When considering price flexibility in the presence of strategic consumers, the value of commitment tends to dominate the advantages of flexibility. Aviv and Pazgal (2008) show that the retailer has an incentive to commit to a fixed pricing strategy over a flexible strategy. While most of this literature shows that a firm should avoid discounting to prevent strategic customer behavior, Elmaghraby et al. (2008) show that a precommitted markdown dominates a single fixed price. Cachon and Feldman (2015) also show that when customers incur search costs, the firm should commit to frequent discounts. Volume flexibility can also be a useful tool to mitigate adverse consumer behavior (see Cachon and Swinney 2009). Yin et al. (2009) show that hiding inventory information from the customers could mitigate some of the customers' strategic response. Lobel et al. (2015) show that committing to a set schedule of product launches is better than having the flexibility to release products over time.

Chod and Rudi (2005) and Chod et al. (2010) argue that flexibility (in pricing or production capacity) is especially important as an instrument to protect the firm against demand variability and correlation. Goyal and Netessine (2011) also show that the value of flexible production capacity depends on the level of demand correlation across different products. In the context of supply chains, Barnes-Schuster et al. (2002) show how flexible contracts with options can further coordinate the supply chain. Anand et al. (2008) show that a dynamic contract is preferred over a committed contract by the supplier, the buyer, and consumers. In this case, the flexible contract empowers the buyer and reduces double marginalization, bringing the system to a higher level of efficiency.

As seen in the literature surveyed above, the value of flexibility is evident from an operational point of view—e.g., matching supply and demand. On the other hand, commitment can be valuable when it encourages a certain behavior from another player. In our context, the efficiency gains of flexibility are typically dominated by the reduced incentives for early production. Under a flexible subsidy policy, the government can get closer to a desired target sales, but the supplier extracts more surplus from the system. Therefore, a committed subsidy policy typically has a lower cost for the government.

1.3. Structure

The remainder of the paper is organized as follows. In Section 2, we present the models for the government and the supplier. In Section 3, we solve the optimization problems and analytically compare the outcomes under the flexible and committed settings. We investigate several extensions of the model in Section 4. We test these results with several computational experiments in Section 5 and provide some concluding remarks in Section 6. All proofs are relegated to the appendix.

2. Model

As we previously mentioned, we consider a dynamic Stackelberg game between the government and the supplier. The government is choosing a subsidy level to offer consumers at each period, denoted by r_t, followed by the supplier, who decides on production quantities u_t. At the end of each period, the uncertain demand is realized, and the remaining inventory (if any) is carried over to the next period. The two settings mentioned before, committed and flexible, differ only on the timing of the government's decision. Under a committed setting, the government sets subsidy levels for all periods before the horizon begins and commits to these subsidies. In the flexible setting, the subsidy levels are decided at the beginning of each period, possibly varying as a function of previous production quantities and realized demand levels.

To keep the analysis tractable and draw insights, we consider a two-period horizon, $t \in \{1,2\}$. The advantages of policy commitment versus flexibility should be evident even within this two-period model. The intuition built for two periods can be expanded for longer horizons, as the different periods decouple given the state of the system—namely, the leftover inventory and the realized sales level. For conciseness, we focus only on the two-period setting.

Within these two time periods, the government aims to achieve an adoption target level Γ, in expectation. More precisely, the government's goal is to incentivize at least Γ consumers to adopt the technology by the end of the time horizon. This policy target is public information, known to consumers and the industry. For example, in his 2011 State of the Union address, U.S. President Barack Obama mentioned the following goal: "With more research and incentives, we can break our dependence on oil with bio-fuels and become the first country to have a million electric vehicles on the road by 2015."[6] Another example of such an adoption target is the one set for solar panels in the California Solar Incentive (CSI) program, which had a stated goal to, by 2016, "install approximately 1,940 MW of new solar generation capacity."[7] Hence, in our model, we optimize the subsidy level to achieve a given adoption target level while minimizing government expenditure.

To achieve this target adoption, the government sets consumer subsidy r_t, for each time period t. Any consumer who purchases the product at that time period will be awarded that subsidy. At each period $t \in \{1, 2\}$, the supplier chooses production quantities u_t as a function of the current level of inventory x_t and the subsidy levels r_t announced by the government. The number of available units to be sold at each period is given by $Supply_t = x_t + u_t$.

Demand for the product at time t is realized as a function of the subsidy levels r_t and the nominal uncertain demand ϵ_t. The random variable ϵ_t represents the intrinsic demand for the product if no subsidy was offered (i.e., $r_t = 0$). This intrinsic demand ϵ_t is assumed to have a probability distribution that is known by both the government and the supplier. Assume that for each additional subsidy dollar in r_t, we obtain an additional b_t units of demand. The value b_t is the demand sensitivity at time t with respect to the subsidy. Demand can be formally defined as $Demand_t = b_t r_t + \epsilon_t$.

The sales level s_t will be determined by the subsidy level, the production decisions of the supplier, and the uncertainty realization. Given a supply level and a demand realization at time t, the number of units sold s_t is the minimum of supply and demand—that is, $s_t = \min(Supply_t, Demand_t) = \min(x_t + u_t, b_t r_t + \epsilon_t)$. The inventory left for the next period can be expressed as $x_{t+1} = x_t + u_t - s_t$.

The objective of the government is to minimize total expected spending while still satisfying the adoption sales target, Γ, in expectation. More precisely, in our two-period model, the government's objective is to minimize $E[Spending] = E[r_1 s_1 + r_2 s_2]$ subject to an average sales target constraint: $E[Sales] = E[s_1 + s_2] \geq \Gamma$.

The subsidy optimization model with an adoption target described above is not the only possible model for the government. For example, one may consider other target constraints on the distribution of sales. Alternatively, the government could maximize sales or social welfare with a budget constraint (see, e.g., Taylor and Xiao 2014, Alizamir et al. 2016). As we show later in our model, the government reacts to early low sales by increasing the subsidy level in later periods. This creates what we call the *undersupply incentive*. Among our main results in this paper, we show that flexibility is typically costlier for the government because of this incentive. Any alternative model for the government problem where the subsidy increases when early sales are low should still create this undersupply incentive for the firm. For simplicity, we focus on the expected sales level constraint model but note that alternative government constraints should yield qualitatively similar results. To gain tractability and isolate the effect of interest (i.e., comparing the time dynamics of the subsidy policy), we ignore the dynamic trajectory effects in our model. More precisely, issues such as learning-by-doing, investments in R&D, and environmental externalities are absent in our model.

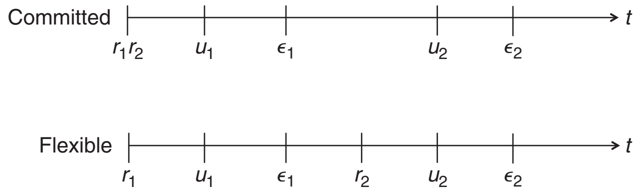
The supplier seeks to maximize the total expected profits by choosing production levels u_t. There is a fixed linear production cost c_t for each unit produced, u_t. The unit selling price p_t is assumed to be exogenous and fixed before the beginning of the time horizon.

Units not sold by the end of the horizon ($t = 2$) get sold for a salvage value denoted by p_3. More formally, the supplier's objective can be written as $E[Profit] = E[p_1 s_1 - c_1 u_1 + p_2 s_2 - c_2 u_2 + p_3 x_3]$. In summary, the two players are solving the following optimization problems.

Government	Supplier
$\min_{r_1, r_2 \geq 0} \; E[Spending]$ $E[Sales] \geq \Gamma$	$\max_{u_1, u_2 \geq 0} E[Profit]$

In this paper, we focus on a single supplier, which can be seen as an aggregate industry player (we explore the case of multiple competing suppliers in Section 4.2). If we assume that there are multiple symmetric suppliers and the aggregate demand is split deterministically across all firms, Lippman and McCardle (1997) show that there is a unique equilibrium to the competitive single-period newsvendor game. Furthermore, this equilibrium is symmetric, and the aggregate order level is the same as the monopolistic setting. Using that same logic in our dynamic

Figure 1. Sequence of Events Under Committed and Flexible Settings



model, all of the results in this paper can be derived for the symmetric competitive setting. Looking at the single supplier as an aggregate industry player further motivates the exogenous price that is not controlled by a given firm. We present the model in this paper using a single supplier to simplify the exposition.

As mentioned before, the order of decisions is the key difference between the two settings we want to study: committed and flexible. In the committed setting, the government commits to subsidy levels r_1 and r_2 for both consecutive periods. The supplier then decides the first production quantity u_1, and the first-period nominal demand ϵ_1 is realized. Observing the amount of inventory x_2 left after the first period, the supplier decides the second production quantity u_2. The second demand ϵ_2 is then realized. In the flexible setting, the government chooses only the first subsidy level r_1. The supplier then follows by choosing a production quantity u_1, and the first-period demand ϵ_1 is realized. At the end of the first period, the government sets the subsidy level for the second period, r_2, followed by the supplier's decision u_2 and the demand realization ϵ_2. The sequence of events describing these two settings is displayed in Figure 1. We use the superscripts c and f to represent the committed and flexible settings, respectively. We next present in more detail the dynamic programs for each setting. In Section 3, we use backward induction to find the subgame perfect equilibrium under each setting.

2.1. Committed Setting

In the committed setting, the government leads the game by choosing both subsidy levels, and the supplier follows by deciding production quantities. As we previously mentioned, the notion of commitment we consider in this paper is a modeling assumption. It is indeed possible that after a change of party in the government, this assumption will not be satisfied anymore. While we do not attempt to prove that governments can commit, we intend to study what would happen to subsidy programs if they could commit. In the committed setting, the optimal decisions by each party can be viewed as a dynamic optimization problem. In the first stage, the government chooses a subsidy policy r_1 and r_2 subject to the optimal production policy set by the supplier. The optimal supplier policy can be expressed as the solution to a two-stage profit maximization problem, for given values of r_1 and r_2.

Let $h_2^c(x_2, r_2, \epsilon_1)$ denote the second-period profit-to-go of the supplier under the committed setting, given the current inventory level x_2 and the demand realization ϵ_1. We do not assume ϵ_1 and ϵ_2 to be independent; therefore, the first-period demand realization is part of the state-space in the dynamic optimization. If demands are independent, the state space would only be (x_2, r_2).

In the first period, the manufacturer solves the following problem to maximize the expected first-period profit plus the profit-to-go for the second period. Note that the effect of the first production decision u_1 on the profit-to-go is captured by the inventory x_2. This quantity is given by the supply level in the first period minus the sales: $x_2 = x_1 + u_1 - \min(x_1 + u_1, b_1 r_1 + \epsilon_1)$. The optimal objective value of this optimization problem is defined as the optimal expected profit of the supplier:

$$h_1^c(r_1, r_2) = \max_{u_1 \ge 0} E_{\epsilon_1}[p_1 \min(x_1 + u_1, b_1 r_1 + \epsilon_1) - c_1 u_1 + h_2^c(x_2, r_2, \epsilon_1)]. \quad (1)$$

At the beginning of the second period, the manufacturer solves problem (2) to maximize the second-period expected profit that includes the remaining salvage value. This problem also defines the profit-to-go function used in the first-period optimization in (1):

$$h_2^c(x_2, r_2, \epsilon_1) = \max_{u_2 \ge 0} E_{\epsilon_2 | \epsilon_1}[p_2 \min(x_2 + u_2, b_2 r_2 + \epsilon_2) - c_2 u_2 + p_3 \max(x_2 + u_2 - b_2 r_2 - \epsilon_2, 0)]. \quad (2)$$

The objective function above is composed of the second-period expected revenue, minus production cost, plus the expected salvage value for leftover inventory at the end of the horizon.

We define $u_1^c(r_1, r_2)$ and $u_2^c(x_2, r_2, \epsilon_1)$ to be the optimal production quantities under the committed setting, which are the optimal solutions of problems (1) and (2), respectively, as a function of the subsidy levels r_1 and r_2. Given the supplier's best-response policy, the government's objective is to minimize the expected spending, subject to a target adoption constraint. The government problem under the committed setting is given by

$$\begin{aligned} E[Spending^c] &= \min_{r_1, r_2 \ge 0} E[r_1 \min(x_1 + u_1^c(r_1, r_2), b_1 r_1 + \epsilon_1) \\ &\qquad + r_2 \min(x_2 + u_2^c(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \quad (3) \\ &\text{s.t.} \quad E[\min(x_1 + u_1^c(r_1, r_2), b_1 r_1 + \epsilon_1) \\ &\qquad + \min(x_2 + u_2^c(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \ge \Gamma. \end{aligned}$$

The optimal solution to problem (3) defines the optimal subsidy levels r_1^c and r_2^c and the optimal expected spending level $E[Spending^c]$ under the committed setting. Note that the resulting subsidy and production levels are a subgame perfect equilibrium, since at every decision point in the timeline described in Figure 1, each player is playing an equilibrium strategy. The expected profit of the supplier is defined as $E[Profit^c] = h_1^c(r_1^c, r_2^c)$. The total sales under the optimal subsidy levels is defined as $Sales^c = \min(x_1 + u_1^c(r_1^c, r_2^c), b_1 r_1^c + \epsilon_1) + \min(x_2 + u_2^c(x_2, r_2^c, \epsilon_1), b_2 r_2^c + \epsilon_2)$.

2.2. Flexible Setting

In the flexible setting, the government leads the game by choosing only the first-period subsidy level. The supplier follows by choosing the production quantity for the first period and then the game is repeated for the second period. The optimal decisions by each party can be viewed as a multitiered optimization problem. In the first stage, the government chooses a subsidy policy r_1 anticipating the optimal response of the supplier, u_1. That production quantity, u_1, is decided by the supplier while considering the government's policy for the second-period subsidy r_2, which is itself a function of the sales in the first period.

From the supplier's perspective, the state of the system at the second period is composed of the leftover inventory, x_2, the subsidy level, r_2, and the demand realization, ϵ_1. Note that ϵ_1 can have some information about the next demand realization ϵ_2, as we consider a correlated demand model. For any given state, define $h_2^f(x_2, r_2, \epsilon_1)$ as the profit-to-go function of the supplier at period $t = 2$ under the flexible setting.

From the government's perspective, the state of the system at the second period is composed of the sales from the first period, s_1, the leftover inventory of the supplier, x_2, and the demand realization, ϵ_1. The first-period sales captures information about how far the government is from its target level Γ. The inventory level affects the possibility of a stockout, and the previous demand realization may influence future demand. Knowing the strategy of the supplier, the government can set the second subsidy level r_2 that minimizes the cost of achieving the remaining target. We denote by $g^f(x_2, s_1, \epsilon_1)$ the second-period cost-to-go of the government.

Note that for tractability purposes, we assume that for the case with demand correlation, the demand realization ϵ_1 is fully observable at period 2 to both the government and the supplier. As mentioned before, in the absence of demand correlation over time, the state-space could be simplified (without ϵ_1).

Because of the sequential nature of the dynamic problem for the flexible setting, we first formulate the optimization problems for the second period:

$$h_2^f(x_2, r_2, \epsilon_1) = \max_{u_2 \ge 0} E_{\epsilon_2|\epsilon_1}[p_2 \min(x_2 + u_2, b_2 r_2 + \epsilon_2) - c_2 u_2 + p_3 \max(x_2 + u_2 - b_2 r_2 - \epsilon_2, 0)]. \quad (4)$$

Define $u_2^f(x_2, r_2, \epsilon_1)$ as the optimal second-period production policy under the flexible setting, which is the optimal solution to problem (4). The government problem in the second period can be written as follows:

$$\begin{aligned} &g^f(x_2, s_1, \epsilon_1) \\ &= \min_{r_2 \ge 0} E_{\epsilon_2|\epsilon_1}[r_2 \min(x_2 + u_2^f(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \\ &\quad \text{s.t.} \quad s_1 + E_{\epsilon_2|\epsilon_1}[\min(x_2 + u_2^f(x_2, r_2, \epsilon_1), b_2 r_2 + \epsilon_2)] \ge \Gamma. \end{aligned} \quad (5)$$

Define $r_2^f(x_2, s_1, \epsilon_1)$ as the optimal second-period subsidy under the flexible setting, which is the optimal solution to problem (5). Knowing the government's future response in subsidy policy, the supplier can decide its first-period production level by solving the following optimization problem:

$$\begin{aligned} h_1^f(r_1) = \max_{u_1 \ge 0} E_{\epsilon_1}[&p_1 s_1 - c_1 u_1 + h_2^f(x_1 + u_1 - s_1, \\ &r_2^f(x_1 + u_1 - s_1, s_1, \epsilon_1), \epsilon_1)], \\ &\text{where } s_1 = \min(x_1 + u_1, b_1 r_1 + \epsilon_1). \end{aligned} \quad (6)$$

Note that we use s_1 as a shorthand notation for first-period sales, which should not be confused as an optimization constraint. The optimal first-period order quantity, $u_1^f(r_1)$, should maximize both the immediate expected profit plus the expected second-period profit-to-go. Knowing the contingent production strategy of the supplier, $u_1^f(r_1)$, the government must then find the optimal first-period subsidy r_1^f that minimizes both the immediate cost and the second-period cost-to-go.

$$\begin{aligned} E[Spending^f] = \min_{r_1 \ge 0} E[&r_1 s_1 + g^f(x_1 + u_1^f(r_1) - s_1, s_1, \epsilon_1)], \\ &\text{where } s_1 = \min(x_1 + u_1^f(r_1), b_1 r_1 + \epsilon_1) \end{aligned} \quad (7)$$

By sequentially solving problems (4)–(7), one can obtain the optimal decision variables for both the supplier and the government under the flexible setting. The expected government spending $E[Spending^f]$ is defined in (7). From (6), we define the supplier's expected profit under the optimal subsidy: $E[Profit^f] = h_1^f(r_1^f)$.

3. Impact on Government and Supplier

In this section, we solve the dynamic programming through backward induction for both the committed and flexible settings and characterize the optimal decision variables. Then, we compare the outcomes in both settings for the government, the supplier, and the consumers.

3.1. Optimal Subsidy and Production Levels

To keep the analysis tractable when solving problems (1)–(7), we impose a few assumptions on the model parameters, which we argue are reasonable for markets with developing technologies. The first assumption relates to demand correlation across time periods. Dynamic games are often studied with independent shocks, but this would remove one of the key benefits of flexibility, which is adapting to new demand information. In this paper, we consider a more general model that allows positive or negative correlation across time periods. In particular, we assume that a random shock from the first-period demand can linearly affect the second-period demand. This model is used in the literature for various applications (see, e.g., See and Sim 2010). The nominal demand model we consider is summarized in the following assumption.

Assumption 1. *Define the nominal demand ϵ_t at time $t \in \{1,2\}$ by*

$$\epsilon_1 = \mu_1 + w_1, \qquad \epsilon_2 = \mu_2 + \alpha w_1 + w_2.$$

$\mu_t > 0$ is the average demand at time t. The random shocks w_1 and w_2 are independent random variables with zero mean: $E[w_1] = E[w_2] = 0$. We denote the cumulative distribution function (cdf) of w_t by the continuous function $F_t(\cdot)$, which is assumed to be common knowledge for both the government and the supplier. In addition, the random variables w_t are assumed to have bounded supports, $w_t \in [A_t, B_t]$, such that the nominal demands are nonnegative—i.e., $\mu_1 + A_1 \geq 0$ and $\mu_2 + \min(\alpha A_1, \alpha B_1) + A_2 \geq 0$.

Note that the cdfs $F_t(\cdot)$ do not need to be identical across time periods. The parameter α represents the level of correlation between time periods (α can be either positive or negative). More precisely, the correlation coefficient between ϵ_1 and ϵ_2 is given by $\mathrm{Corr}(\epsilon_1, \epsilon_2) = \alpha\sqrt{\mathrm{Var}(w_1)/\mathrm{Var}(w_2)}$.

Note also that we make an implicit assumption that customers are not strategic. In particular, customers who arrive in the first period are either not forward looking or simply different customers than the ones that arrive in the second period. Apart from the correlation factor α, there is nothing that links the demand across the two periods. This assumption is used primarily for tractability, since incorporating the strategic timing of customers can lead to a much more complicated analysis (see, e.g., Alizamir et al. 2016).

In early stages of the introduction of new technologies, it is often common to observe decreasing prices and costs over time. In addition, profit margins are often decreasing over time, as additional players are entering the market. With this in mind, we restrict our analysis with the following set of inequalities summarized in Assumption 2. Note that in our model, we assume that the supplier is a price-taker, so that p_1, p_2, and p_3 are exogenous market prices (p_3 being the salvage value at the end of the horizon). The price effect on demand is captured by μ_t, $t = 1, 2$. The marginal costs of production are denoted by c_1 and c_2.

Assumption 2. *We make the following assumptions on prices, costs, and profit margins:*

1. *Prices and costs are decreasing over time—i.e., $p_1 > p_2$ and $c_1 > c_2$.*
2. *Profit margins are positive and decreasing—i.e., $p_1 - c_1 > p_2 - c_2 > 0$.*
3. *Salvage value is smaller than production cost: $c_2 > p_3$.*

Decreasing prices and costs are commonly observed in the literature for new product introduction. Lobel and Perakis (2017), for instance, surveys the literature on the declining costs of solar photovoltaic technology, mostly attributed to learning effects. Lee et al. (2000) show additional evidence of declining prices in the PC industry within the product life cycle. Note that cost decreases are often attributed to learning-by-doing, which could be modeled endogenously as a function of units sold or produced. As we will see later, the committed setting already has an advantage to encourage higher supply levels. In this case, endogenous learning might give further advantage to the committed setting. To simplify the problem and to focus solely on the impact of the game dynamics, we assume that the production cost reduces exogenously.

For the same reason, we restrict our model to the case with decreasing profit margins. If profit margins were to increase, we would provide further incentives for the supplier to delay production. The production delay would be more accentuated in the flexible setting, making a stronger case for policy commitment.

We next define in Table 1 a set of quantiles of the cumulative distribution of demand uncertainty, $F_t(\cdot)$. We later show in Lemma 1 that these quantities represent the optimal production quantiles of the supplier in the different periods and settings.

Note that the production quantiles for the second period are the same in both setting $k_2^c = k_2^f = k_2$. In addition, observe that $k_1^f \leq k_1^c$. Before showing the optimality of the production quantiles from Table 1, we impose an additional assumption. More precisely, we restrict our attention to the case where the supplier does not stay idle at any of the time periods. This happens when the leftover inventory is smaller than the desired supply level for the next period. Otherwise, the optimal ordering policy would have a discontinuity that makes the problem analytically intractable in the first period. Realistically, green technology products are expensive to manufacture and typically do not face a critical oversupply where the leftover inventory from one year covers all demand for the next year. For this reason, we restrict the magnitude of the demand noise so that the inventory x_2 should be no larger than the desired supply level at period 2 for any realization of w_1. We also restrict our attention to the case where the adoption

Table 1. Production Quantiles

Committed	Flexible
$k_1^c = F_1^{-1}\left(\dfrac{p_1 - c_1}{p_1 - c_2}\right)$	$k_1^f = F_1^{-1}\left(\dfrac{(p_1 - c_1) - (p_2 - c_2)}{p_1 - p_2}\right)$
$k_2^c = F_2^{-1}\left(\dfrac{p_2 - c_2}{p_2 - p_3}\right)$	$k_2^f = F_2^{-1}\left(\dfrac{p_2 - c_2}{p_2 - p_3}\right)$

target cannot be reached without the presence of government subsidies. We summarize this discussion in the following assumption.

Assumption 3. *On the magnitude of demand uncertainty and adoption target*:

1. *Desired supply at* $t = 1$ *is always larger than initial inventory—i.e.,* $k_1^f + \mu_1 \geq x_1$.
2. *Desired supply at* $t = 2$ *is always larger than leftover inventory—i.e.,* $k_2 + \mu_2 \geq k_1^c - A_1 - \min(\alpha A_1, \alpha B_1)$.
3. *The adoption target is large enough—i.e.,* $\Gamma \geq 2(E[\min(k_1^c, w_1)] + \mu_1)$ *and* $\Gamma \geq 2(E[\min(k_2^c, w_2)] + \mu_2)$.

Assumption 3 is not necessary, but is sufficient to guarantee that the supplier will not idle. Note that in the first part, we use k_1^f, whereas in the second part, we use k_1^c. Since $k_1^f \leq k_1^c$, this ensures that both conditions are satisfied under both settings. The first part, $k_1^f + \mu_1 \geq x_1$, guarantees that the first-period production level is nonnegative. Indeed, if the initial inventory is too large, the problem becomes uninteresting. The second part means that the target "newsvendor" service level of the second period is larger than in the first period. In other words, in the absence of a subsidy policy, the manufacturer would try to serve a larger number of customers in the second period simply from demand, cost, and price conditions. The last part of the assumption ensures that the government subsidy policy is actually needed to meet the target adoption. In other words, we want to restrict our model with Assumption 3 to ensure that $r_t^j > 0$ and $u_t^j > 0$ for any period t and setting j.

Under Assumptions 1–3, one can obtain the optimal production policies for the supplier in each setting. The results are derived in closed form and summarized in Lemma 1.

Lemma 1. *The optimal ordering production and subsidy levels for both settings are given by*

Committed

$$u_1^c(x_1, r_1) = b_1 r_1 + k_1^c + \mu_1 - x_1,$$
$$u_2^c(x_2, r_2, w_1) = b_2 r_2 + k_2 + \mu_2 + \alpha w_1 - x_2,$$
$$r_1^c = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^c + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2 + \mu_2}{2(b_1 + b_2)},$$
$$r_2^c = \frac{\Gamma}{b_1 + b_2} - \frac{(v_2 + \mu_2)(b_1 + 2b_2)}{2b_2(b_1 + b_2)} - \frac{v_1^c + \mu_1}{2(b_1 + b_2)};$$

Flexible

$$u_1^f(x_1, r_1) = b_1 r_1 + k_1^f + \mu_1 - x_1,$$
$$u_2^f(x_2, r_2, w_1) = b_2 r_2 + k_2 + \mu_2 + \alpha w_1 - x_2,$$
$$r_1^f = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^f + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2 + \mu_2}{2(b_1 + b_2)},$$
$$r_2^f(s_1, x_2, w_1) = \frac{\Gamma - s_1 - \mu_2 - \alpha w_1 - v_2}{b_2}$$

Note that the above ordering quantities are functions of the subsidy levels as they are computed as best responses. Note also that the optimal supply level at time t, $u_t + x_t$, is expressed as the nominal demand level, plus the demand boost from the subsidy $b_t r_t$, adjusted by the newsvendor quantile k_t. With this optimal production policy, one can solve the government optimization problem and obtain the optimal subsidy policy. To simplify the notation, we denote by v_t^j the expected demand uncertainty truncated by the optimal quantile. That is, $v_t^j = E[\min(k_t^j, w_t)]$, for setting $j \in \{c, f\}$ and time period $t \in \{1, 2\}$. Note that $v_2^f = v_2^c = v_2$.

One can see that the optimal subsidy levels for the second period under the flexible setting $r_2^f(s_1, x_2, w_1)$ is a random variable that depends on the realization of w_1. For comparison purposes, it can be useful to compute the corresponding expected value. We first compute the expected sales at the first period: $E[s_1] = b_1 r_1^f + \mu_1 + v_1^f$. Consequently, the expected subsidy level for the second period in the flexible setting is given by

$$E[r_2^f(s_1, x_2, w_1)] = \frac{\Gamma}{b_1 + b_2} - \frac{(v_2 + \mu_2)(b_1 + 2b_2)}{2b_2(b_1 + b_2)} - \frac{v_1^f + \mu_1}{2(b_1 + b_2)}.$$

3.2. Comparisons

When comparing the flexible and committed settings, the first thing to notice is the difference in the optimal quantiles for any given subsidy levels. To provide further intuition for the optimality of the ordering quantiles described in Table 1, one can look at the cost of underage and overage in traditional newsvendor models. Note that the key difference between k_1^c and k_1^f is the cost of undersupplying the market demand. In the single-period newsvendor model, the costs of underage (C_u) and overage (C_o) define the optimal ordering quantile as $F^{-1}(C_u/(C_u + C_o))$. Since there is no idling in the second period, an additional unsold unit (overage) will incur a cost that is simply the difference in production cost over time. For both settings, the cost of overage is defined as $C_o = c_1 - c_2$. For the committed setting, the underage cost is defined by the opportunity cost, or profit margin forgone, $C_u^c = p_1 - c_1$. In the flexible case, an unmet unit of demand will be compensated by an equivalent unit of demand from increased rebates in the second period. Therefore the underage cost is the difference in profit margins, $C_u^f = (p_1 - c_1) - (p_2 - c_2)$. Note that the quantiles of Table 1 are also defined by the rule $C_u/(C_u + C_o)$. The proof of Lemma 1 contains a formal proof for this optimality result. Nevertheless, this explanation brings a very interesting intuition: government flexibility reduces the underage risk for the supplier.

This key difference in the ordering levels is further described in Proposition 1. It drives disparities in production, subsidies, and sales in the two settings. Note that sales at a given period t are defined as $s_t = \min(x_t + u_t, b_t r_t + \epsilon_t)$ and is a random variable. With the structure of the optimal production policy defined in Lemma 1, note that sales for each setting $j \in \{c, f\}$ can be simplified to $s_1^j = b_1 r_1^j + \mu_1 + \min(k_1^j, w_1)$ and $s_2^j = b_2 r_2^j + \mu_2 + \alpha w_1 + \min(k_2^j, w_2)$. The following proposition summarizes these comparisons.

Proposition 1. *Comparing production quantiles, expected productions, subsidy levels, and sales between committed and flexible settings*:

- *In the first period, the supplier's optimal production quantile is larger in the committed setting than in the flexible setting—i.e.,* $k_1^c \geq k_1^f$. *In the second period, the quantiles are equal*: $k_2^c = k_2^f = k_2$.
- *The expected production is larger in the first period and lower in the second period in the committed setting. The total expected production is the same in both settings—i.e.,* $u_1^c + E[u_2^c] = u_1^f + E[u_2^f]$.
- *Expected subsidy levels in each period are lower in the committed setting.*
- *The expected sales are higher in the first period with commitment, but are lower in the second period. Also, the total expected sales meet the government target in both settings—i.e.,* $E[s_1^c + s_2^c] = E[s_1^f + s_2^f] = \Gamma$.

Production	*Subsidy*	*Sales*
$u_1^c \geq u_1^f$	$r_1^c \leq r_1^f$	$E[s_1^c] \geq E[s_1^f]$
$E[u_2^c] \leq E[u_2^f]$	$r_2^c \leq E[r_2^f]$	$E[s_2^c] \leq E[s_2^f]$

Note that the subsidy levels and the production quantities in the first period are not random variables. The fact that the total expected sales are equal to the target adoption level is not surprising, as the government uses this condition to derive the optimal solution. Proposition 1 shows that a larger proportion of the target is satisfied in the first period under commitment. To show this, one can calculate the difference in sales quantity: $E[s_1^c - s_1^f] = (b_2/(2(b_1+b_2)))(v_1^c - v_1^f)$. This measure quantifies the average amount of sales that is postponed to the second period when the game dynamics is changed from a committed to a flexible setting.

To understand the effect of this postponement on the total government spending, we need to further analyze the optimal subsidy levels. Using the results from Proposition 1, one can compare the expected level of spending from the government. Under a committed setting, the spending will be given by $E[Spending^c] = E[s_1^c]r_1^c + E[s_2^c]r_2^c$, as subsidy levels are set in a deterministic way. Under a flexible setting, the spending is defined as $E[Spending^f] = E[s_1^f]r_1^f + E[s_2^f r_2^f]$. Note that the subsidy for the second period under the flexible setting is now a random variable and therefore cannot be taken outside the expectation. We next derive the expected total spending levels for the government under the two settings.

Theorem 1. *The expected government spending is given by*

$$E[Spending^c] = (b_1 r_1^c + v_1^c + \mu_1) r_1^c + (b_2 r_2^c + v_2^c + \mu_2) r_2^c$$

$$E[Spending^f] = (b_1 r_1^f + v_1^f + \mu_1) r_1^f + (b_2 E[r_2^f] + v_2^f + \mu_2) E[r_2^f] + \frac{\mathrm{Var}(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 \min\{k_1^f, w_1\}]}{b_2}.$$

The difference in expected spending between the two settings can be written as

$$E[Spending^f - Spending^c] = \overbrace{\frac{1}{4b_1(b_1+b_2)}[2b_1(v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2(v_1^c + \mu_1)^2 - b_2(v_1^f + \mu_1)^2]}^{\text{supplier incentive effect}} + \underbrace{\frac{\mathrm{Var}(\min\{k_1^f, w_1\})}{b_2}}_{\text{adaptability effect}} + \underbrace{\frac{\alpha E[w_1 \min\{k_1^f, w_1\}]}{b_2}}_{\text{correlation effect}}.$$

Corollary 1. *If* $\alpha \geq 0$, *the expected spending is smaller in the committed setting relative to the flexible setting—i.e.,* $E[Spending^c] \leq E[Spending^f]$.

Note that the difference in spending between committed and flexible is derived by different effects, which we label *supply incentive*, *adaptability*, and *correlation*. The first term is induced by the decreased production quantile, $k_1^f < k_1^c$. Since the government flexibility reduces the firm's potential loss from undersupply, we label this as the *supply incentive effect*. This effect captures the cost of the government flexibility to compensate the reduced supply.

The second term, *adaptability effect*, captures the average premium paid by the government for the benefit of adjusting the rebates in the second period. Even in the absence of the supply incentive effect, $k_1^f \to k_1^c$, and the correlation effect, $\alpha = 0$, the adaptability effect on the flexible spending will remain solely because of the volatility in the first-period demand. This effect occurs because a low first demand induces an increase for both the second-period subsidy and the sales. When the first-period demand is high, the positive upside is curbed by the limited supply level k_1^f. This adaptability will effectively "buy" the government a lower variance of the sales, as shown in Theorem 2.

The third term, *correlation effect*, appears when there is intertemporal correlation in demand, $\alpha \neq 0$. Note that the first two effects are always positive. When correlation is nonnegative, Corollary 1 shows that since the third effect is also positive, the committed spending is on average smaller. When α is sufficiently negative, the correlation effect can become the dominant factor and make the expected flexible spending lower relative to the committed setting. This instance is demonstrated in the computational experiments of Section 5.

It should be noted that when the industry player is myopic, the ordering quantiles revert to the classical single-period newsvendor quantiles $F_{w_t}^{-1}(1 - c_t/p_t)$ for both the committed and flexible settings. Therefore, $v_1^f = v_1^c$, which makes the supplier incentive effect vanish. The difference between the spending levels in the two settings is then primarily driven by the adaptability and correlation effects. The identical argument can be made when there is ample supply. Consequently, the flexible policy is typically more expensive (on average) even without the issue of undersupply risk. The strategic nature of the industry player and its interaction with the government only amplify this outcome.

Recall that we assume that the prices and costs are decreasing over time (i.e., $p_1 > p_2$ and $c_1 > c_2$), which is a typical assumption for new technologies. This usually induces a boost in demand in the second period—i.e., $\mu_2 > \mu_1$. Interestingly, the result of Theorem 1 holds even when $\mu_2 > \mu_1$, as long as the supplier is not idle in the first period (see Assumption 3).

One can derive the same insight by looking at the expected government spending for each time period separately (the proof of Corollary 2 resembles the proof of Corollary 1 and is not presented because of space limitations).

Corollary 2. *The per-period expected government spending satisfies the following*:

- *First period*:

$$E[Spending_1^c] \leq E[Spending_1^f].$$

- *Second period*: *If* $\alpha \geq 0$, *then*

$$E[Spending_2^c] \leq E[Spending_2^f].$$

The following result compares the variance of total sales realized under the flexible and the committed setting. Note that the expected sales in both cases equal the adoption target. We show in Theorem 2 below that the total output of sales is more variable under the committed setting.

Theorem 2. *The variance of the sales is larger in the committed setting relative to the flexible setting—i.e.,* $\mathrm{Var}(Sales^c) \geq \mathrm{Var}(Sales^f)$.

In other words, the premium paid for adaptability in expected spending provides a lower variance of the sales. The flexible government will typically reach a final adoption level closer to the desired target. This result holds for any level of demand correlation. Note that this is not variance in spending, which can be significantly more complicated to compare analytically. We do compare the variance in spending computationally in Section 5, where we show that there is no clear dominance between the two settings. In fact, we show how it depends on the market conditions.

In the absence of correlation, the variance in sales and expected spending characterizes a risk–reward trade-off for the government. Depending on how close they want to be to the adoption target, the government might consider paying the premium for a flexible policy.

So far, we have shown that the flexible policy generally yields smaller government expected spending (for each period), while providing a lower variance of the sales (and, hence, providing a higher confidence in the target adoption). However, as we show in Proposition 1, the expected sales (and the expected production) in the first period are higher under the commitment policy. Consequently, if the learning and externality implications of early adoptions are significant, then the committed policy may be preferred. Otherwise, the flexible policy seems to be a better option.

Next, we compare the expected supplier's profit in both settings.

Theorem 3. *The expected profit of the supplier is smaller in the committed setting relative to the flexible setting—i.e.,* $E[Profit^c] \leq E[Profit^f]$.

Theorem 3 states that the supplier will always profit more under a flexible government. This result follows from the lower undersupply opportunity cost of the govenrment in the flexible setting. This lower undersupply cost will lead to a lower production quantile k_1^f, which in turn induces higher average subsidy levels.

It should be noted that this is not a direct manipulation of the government policy by the supplier. In the absence of demand uncertainty or when $p_2 - c_2 \ll p_1 - c_1$, the flexible subsidy level converges to the committed level and the profit difference goes to zero. The game dynamics of the flexible setting does not provide an additional profit for the firm by itself. Nor does the firm hold any informational advantage over the government. The additional profit in the flexible setting comes from the undersupply incentive created by the government policy that boosts demand in the second period if initial sales are low.

3.3. Consumer Surplus

In a deterministic demand model, consumer surplus is typically defined as $D^2/(2b)$. The definition in Equation (8) adjusts for the fact that there is a stockout probability that affects the utility of consumers. The underlying assumption is that every consumer has the same probability of not being served, independent of his or her individual valuation for the product. For more details, see, for example, Cohen et al. (2016) for a definition of consumer surplus under stochastic demand. We define the consumer surplus for one time-period as

$$CS(\epsilon) = \frac{D(\epsilon)\min(u + x, D(\epsilon))}{2b}. \tag{8}$$

Here, u represents the production quantities and x the leftover inventory from the previous period. The total consumer surplus is obtained as the sum of each period's consumer surplus:

$$CS = CS_1(\epsilon_1) + CS_2(\epsilon_2),$$

$$CS = \frac{(b_1 r_1 + \epsilon_1)\min(u_1 + x_1, b_1 r_1 + \epsilon_1)}{2b_1} + \frac{(b_2 r_2 + \epsilon_2)\min(u_2 + x_2, b_2 r_2 + \epsilon_2)}{2b_2}.$$

Note that the consumer surplus is a random variable that depends on both noises. To compare both settings, we look at the expected value of the consumer surplus. In particular, we want to know when consumers are better off in the flexible setting and the key factors. We focus on the uncorrelated case for simplicity—i.e., $\alpha = 0$.

We have shown that in the flexible setting, the subsidies are higher in both periods in expectation. Therefore, the consumers are receiving more money on average per unit sold. In addition, the expected total productions are the same. Therefore, one might naively think that consumers are always better off in the flexible policy. However, this is not always the case, and the expected total consumer surplus can be larger in the committed setting under some conditions. The results are summarized in the following theorem.

Theorem 4. *The expected consumer surplus satisfies the following*:

1. *In the second period, the consumers are always better off in the flexible setting—i.e.,*

$$E[CS_2^f] \geq E[CS_2^c]. \tag{9}$$

2. *In the first period, consumers can be better off or worse off in the flexible setting, depending on the ratio of price sensitivities. In particular, if* $\Gamma \geq 2w_1$, *there exists a threshold value of* b_1/b_2 *above* (*below*) *which the consumers are better off in the flexible* (*committed*) *setting.*

The consumers are mainly affected by the amount of subsidies offered by the government and by the total sales (availability of product). We note that in the second time period, both the expected subsidies and sales are larger in the flexible setting. Consequently, overall it benefits consumers, and the result in (9) is intuitive. On the other hand, the impact on consumers in the first period is more complicated. Indeed, the subsidies in the flexible setting are higher, but the expected sales are lower. As a result, the effect on consumers depends on the trade-off between these two factors. In particular, we show that it depends on the price sensitivity parameters ratio b_1/b_2. If this ratio is large enough, consumers are better off in the flexible setting; if this ratio is small enough, consumers are worse off. The assumption $\Gamma \geq 2w_1$ is a sufficient condition for the existence of the b_1/b_2 threshold. This is a technical nonrestrictive condition, as it only ensures that the target adoption level set by the government cannot be attained simply by a large noise realization.

Finally, one can expect the total expected consumer surplus, $E[CS_1] + E[CS_2]$, to behave in a similar way as the expected consumer surplus in the first period. In particular, there exists a threshold value of b_1/b_2 above which the consumers are better off in the flexible setting (or worse below the threshold).

4. Extensions

In this section, we investigate several extensions of the model and results presented in Sections 2 and 3. More precisely, we consider (i) a government that maximizes welfare (instead of minimizing the expected spending), (ii) a model with competitive suppliers, (iii) a model that accounts for backorders (as opposed to lost sales), (iv) different types of settings leading to several versions of the commitment/flexible regimes, and (v) incorporation of a discount factor. In all five extensions, we show that our main results and insights are preserved.

4.1. Maximizing Welfare

Considering the case where the government maximizes social welfare is an important extension for checking the robustness of the results presented in this paper. Below, we show that our results in terms of comparing the committed and flexible policies still hold when the government maximizes a quadratic social welfare.

Similar results and insights will generally hold when either (i) the adoption constraint is tight at optimality; or (ii) the government objective function ties the sales in the two periods. In other words, the subsidy decision in the second period should depend on the level of adoption from the first period: If the sales are low (high) in the first period, the government would want to increase (decrease) the subsidy in the following period. If this basic dynamic applies, the main insight of our paper should still hold—i.e., commitment encourages higher supply, leading to lower government spending. The commitment policy becomes similar to the flexible policy only if the time periods are completely independent from the government perspective, which is not realistic based on previously mentioned examples of subsidy programs (e.g., Germany and California). Note that for some government objectives, the constraint will not be tight at optimality (i.e., the optimal unconstrained solution satisfies the constraint). If this is the case, and in addition, the sales in the two periods are not linked, the problem becomes less interesting. Note also that when the objective function of the government is monotone with respect to the subsidy level, it is true that the optimal solution will be such that the adoption constraint is exactly met. In this case, we will typically observe the same intuition, since the optimal solution is driven by the adoption constraint. More precisely, a lower adoption in the first period will induce the government to compensate and increase the rebate in the second period. This happens for example when the government minimizes the expected spending or maximizes an increasing welfare function (with respect to the rebates).

In this section, we consider the case where the government maximizes the total welfare without any adoption constraint (or equivalently, the case with a constraint that is not binding at optimality). In this case, we study a common welfare function from the literature and derive the optimal solution, allowing us to show that the same results still hold.

Based on the literature (see, e.g., Cohen et al. 2017, Raz and Ovchinnikov 2015), we use the following welfare definition:

$$Welfare = SupplierProfit + ConsumerSurplus - GovernmentSpending + Externality. \quad (10)$$

The last term captures the social and environmental benefits of additional technology adoption. For instance, if more solar panels are adopted, society will benefit from a cleaner source of energy. As a result, the *Externality* should naturally be increasing with the total level of adoption. It should also be noted that the marginal benefit of adoption is typically decreasing. For instance, too many solar panels can be detrimental to the functioning of the power grid. In addition, early adopters usually have a larger marginal impact than late adopters. For these reasons, we model this externality as a concave function of the total sales. For illustrative purposes and to keep the analysis tractable, we consider a quadratic function of the total sales defined by the parameters $\theta > 0$ and $\lambda > 0$:

$$Externality = \theta\, Sales - \lambda\, Sales^2. \quad (11)$$

Our goal is to solve for the subsidy policies that maximize the unconstrained social welfare measure in (10). In particular, we want to compare the committed and flexible policies and extend the results of the paper to the case where the government maximizes welfare. For simplicity, we assume that the initial inventory is zero (i.e., $x_1 = 0$), the correlation between time period is zero (i.e., $\alpha = 0$), and the salvage value after period 2 is also zero (i.e., $p_3 = 0$). (However, the analysis extends if one relaxes these assumptions, at the expense of a heavier notation.)

When the government is maximizing welfare, we show that the following results hold (the proofs follow a similar methodology as in the original model).

Proposition 2. *Consider the model when the government maximizes the total welfare function in (10). Then, by comparing the committed and the flexible policies, we have the following results*:
- $k_1^c \geq k_1^f$ *and* $k_2^c = k_2^f$.
- $r_1^f \geq r_1^c$ *and* $E[r_2^f] \geq r_2^c$.
- $E[s_1^c] \geq E[s_1^f]$ *and* $E[s_2^c] \leq E[s_2^f]$.
- $u_1^c \geq u_1^f$ *and* $E[u_2^c] \leq E[u_2^f]$.
- $E[Profit^c] \leq E[Profit^f]$.

In addition, the expected spending satisfies $E[Spending^c] \leq E[Spending^f]$, *when the value of* λ *is small enough—i.e., there exists a positive threshold such that the above inequality holds when* λ *is lower than this threshold.*

Finally, we observed by extensive numerical tests that $\mathrm{VAR}[Sales^c] \geq \mathrm{VAR}[Sales^f]$—i.e., the variance in sales is larger under the committed setting.

Note that for the results to be interesting, the government's problem in the second period needs to have some relation to the adoption level of the first period, either through the objective function or via a joint constraint. If both time periods are separable—for instance, if there are no adoption constraints and the welfare function is linear (i.e., $\lambda = 0$), the committed and the flexible settings coincide, and hence this case would not be as interesting. On the other hand, with a quadratic welfare function and any value of $\lambda > 0$, our general results hold. Alternatively, if there is a joint constraint that connects the adoption between both periods, our results hold too (even when $\lambda = 0$). These two alternatives exemplify different modeling approaches that capture how governments react to past sales in future policy decisions. In essence, our results are driven by the fact that a low demand scenario in the first period is compensated by a higher subsidy in the second period, under a flexible setting. We believe that any monotonically increasing concave welfare functions may also qualitatively behave in this way, as it induces decreasing marginal returns to subsidies. Consequently, the flexible government will still adjust so as to compensate the adoption trajectory.

4.2. Competitive Suppliers

In this section, we extend the analysis for the case with several competing suppliers. We first convey the fact that the results from Lippman and McCardle (1997) still hold in a two-time-period setting, based on Caro and Martínez-de Albéniz (2010). Second, we show that for the committed setting, having a monopolist is equivalent to considering competing firms (under some assumptions detailed in the appendix). However, this is not correct for the flexible setting. In particular, adding competition reduces the undersupply incentive effect. Nevertheless, we show that the same effect is still present: adding an additional supplier will attenuate the difference between the committed and the flexible policies, but the same qualitative effect is preserved. For a duopoly of suppliers, we can show that the flexible policy still induces an undersupply incentive that yields a larger expected spending for the flexible policy. This suggests that the monopolist assumption can be used as a simple stylized model to capture and study our problem. Note that based on the literature above, these results might be generalized to any finite number of players.

In the appendix, we present the extension of the result from Lippman and McCardle (1997) to two time periods by considering the case where the firms face an aggregate industry-wide shock (based on the work by Caro and Martínez-de Albéniz 2010). We conclude that the two suppliers will either stock out together or have excess inventory together. Note that the result is proved for a setting with suppliers that have symmetric prices and costs. Nevertheless, it still allows the firms to have different initial inventories and to receive unequal shares of the uncertain demand. A direct conclusion is the fact that the subsidies and aggregate industry production for the competitive committed setting are exactly the same as in the monopolist model (i.e., when the suppliers merge).

In the flexible setting, the government adjusts the subsidies dynamically. Thus, the timing of the decisions may affect the production of the two competing suppliers. The main insight we demonstrate next is that the undersupply incentive is preserved—i.e., the flexible policy of the government induces lower supply from the firms in the first period, $k_1^f \leq k_1^c$. Note that in the second period, using the same argument as in the committed setting, one can show that for each supplier $k_2^f = k_2^c$, and these are exactly the same expressions as in the monopolist setting. Nevertheless, we obtain a different story in the first time period.

Recall that in the monopolist setting, we have the following:

$$k_1^f = F^{-1}\left(\frac{(p_1 - c_1) - (p_2 - c_2)}{p_1 - p_2}\right) \leq k_1^c = F^{-1}\left(\frac{p_1 - c_1}{p_1 - c_2}\right). \quad (12)$$

In the setting with two competing suppliers, we obtain the following production quantiles, which still follow the same relationship.

Proposition 3. *Consider the setting with two competing suppliers, under the aforementioned assumptions. Then, the production quantile for each supplier $i = 1, 2$ follows*:

$$k{f}_1^i = F^{-1}\left(\frac{(p_1 - c_1) - q^i(p_2 - c_2)}{(p_1 - c_2) - q^i(p_2 - c_2)}\right) \leq k_1^c = F^{-1}\left(\frac{p_1 - c_1}{p_1 - c_2}\right). \quad (13)$$

Here, $0 \leq q^i \leq 1$ represents the portion of the demand that supplier i receives ($q^1 + q^2 = 1$). The proof follows a very similar methodology as in the monopolist case and is omitted for conciseness. Note that when $q^i = 1$, the expression coincides with the monopolist setting. For any $0 < q^i < 1$, one can see that $k{f}_1^i$ is smaller than k_1^c. However, $k{f}_1^i$ with two suppliers is larger relative to k_1^f for a monopolist. Consequently, the undersupply incentive reduces as the industry becomes competitive.

Using the production quantile from Equation (13), one can solve for the optimal subsidy r_1^f, r_2^f. Replicating the same procedure as in the monopolist setting with the new adjusted quantile from Equation (13), we observe that the results of the paper (comparison of rebates, production, expected government spending, and profits of the supplier) are preserved.

In conclusion, we have shown that considering a model with a single supplier allows us to study the difference between the committed and flexible settings. In other words, under some common assumptions, the results and insights of this paper can be generalized to incorporate competition.

4.3. Model with Backorders

We next extend our analysis and results to a model that allows for backorders (as opposed to lost sales). Interestingly, our results still hold; hence, this strengthens the key message of our paper. In particular, Theorem 1 and Corollary 1 still hold. We first show that in a model with backorders, the undersupply incentive in the first period vanishes—i.e., $k_1^c = k_1^f$. Consequently, the supplier's incentive to underproduce in the first period under the flexible regime disappears. That being said, we can show that Theorem 1 still holds—i.e., the expected spending is higher in the flexible setting, assuming that the correlation factor $\alpha \geq 0$. Even though the supplier incentive effect vanishes, the adaptability and correlation effects are still present.

In the original model, we assumed that unmet demand in the first period was lost. Namely, if there is not enough supply when the customer shows up, the customer will leave the market. We next extend our analysis and results to a model that allows for backorders. In this case, if a customer does not purchase in the first period because of the lack of supply, the unmet demand will reappear in the second period. Mathematically, we represent backorders by allowing the state variable x_2 to be negative—i.e., if x_2 is positive, we have excess supply, and if x_2 is negative, we have excess demand. More precisely, we define the negative part $x_2^- = \min(x_2, 0)$ as the level of backordered demand in absolute terms. Similarly, the positive part $x_2^+ = \max(x_2, 0)$ is the leftover inventory, as in the previous model. Note that holding inventory or backorders are mutually exclusive. The state of the system can then be expressed as $x_2 = x_2^+ + x_2^-$. For simplicity, we assume there is no salvage value in the following analysis, $p_3 = 0$. We next consider both the committed and flexible settings for the model with backorders and compare the optimal decision variables.

By sequentially solving the optimization problems of the supplier and the government, in a similar fashion as in Section 3, one can derive the optimal decision variables. We next compare the optimal decision variables as well as the expected government spending in both the committed and flexible settings.

Proposition 4. *Consider the model with backorders. Then, when comparing the flexible and committed settings, we have the following.*

- *The production quantiles are equal*: $k_1^c = k_1^f$ *and* $k_2^c = k_2^f$.
- *The optimal ordering quantities satisfy* $u_1^c = u_1^f$ *and* $E[u_2^f] = u_2^c$.
- *The optimal subsidy levels satisfy* $r_1^c = r_1^f$ *and* $E[r_2^f] = r_2^c$.
- *The total expected profit is the same*: $E[Profit^c] = E[Profit^f]$.
- *The difference in expected spending between the two settings can be written as*

$$E[Spending^f] - E[Spending^c] = E[w_1 \min(k_1, w_1)](1 + \alpha)/b_2.$$

The proof of Proposition 4 is in the same spirit as the case with lost sales and is omitted for conciseness. Note that in the model with backorders, we have $k_1^c = k_1^f$ and, therefore, the supplier incentive to underproduce in the first period under the flexible regime disappears. Thus, all of the optimal decision variables are equal in expectation. However, since r_2^f is a random variable (as opposed to r_2^c that is deterministic), the adaptability effect is still present. In addition, if $\alpha \geq 0$, the expected spending is smaller in the committed setting relative to the flexible setting—i.e., $E[Spending^c] \leq E[Spending^f]$.

This shows that the main result of our paper still holds in the model with backorders. Even though the undersupply incentive of the supplier disappears, the flexible policy still induces an adaptability effect and a correlation effect that translate to a higher expected spending.

4.4. Alternative Time Dynamics

When comparing the committed and flexible policies, the two settings did not impose the same adoption constraint. More precisely, the flexible policy imposed a stricter constraint that requires the adoption constraint to be satisfied for any given realization of the first-period sales. In this section, we consider two alternative settings, *Robust* and *Semiflexible*, that address this issue. In the *Robust* setting, the government must commit to a policy, but subject to a constraint that holds for every possible realization of ϵ_1. In the *Semiflexible* setting, the government must decide on an adjustable policy $r_2(s_1)$ before the realization of ϵ_1, subject to an expected target constraint over both periods. We then compare these new settings with the original specifications of the paper to show how the changing constraints influence the results. Below, we present both new approaches.

1. *Robust Policy.* The first approach is to tighten the target adoption constraint under the committed policy—i.e., requiring the target Γ to be met for every possible realization of the noise ϵ_1 in the first period. This is equivalent to imposing the adoption constraint to be met for the worst-case (lowest) realization; hence, we call this setting the robust policy. Note that in this case, the timing of the decisions is the same as in the committed setting. The only difference is that instead of looking at every possible scenario for the random variable ϵ_1, we only look at the worst-case scenario for the demand in the first period—i.e., the lowest realization of ϵ_1. We next compare the robust policy to the committed and flexible settings, by extending all of the results (i.e., subsidy levels, expected government spending, and variance of the sales). For simplicity, we assume that there is no correlation factor between the time periods—i.e., $\alpha = 0$.

Proposition 5. *Consider the three settings*: *committed, flexible, and robust* (*denoted by the superscripts c, f, and r, respectively*). *Then, we have the following*:

- *The optimal subsidy levels of the first and second time periods follow*

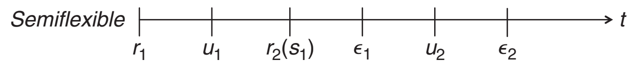
$$r_1^c \leq r_1^f \leq r_1^r,$$
$$r_2^c \leq E[r_2^f] \leq r_2^r.$$

- *The expected government spending satisfies the relation* $E[Spending^c] \leq E[Spending^f] \leq E[Spending^r]$.
- *The variance of the sales follows* $\mathrm{Var}(s^c) \geq \mathrm{Var}(s^f) = \mathrm{Var}(s^r)$.

Consequently, we have extended the results of the paper for the robust policy. Since the adoption constraint is tighter, it is clear that the robust policy will be more costly relative to the committed setting that imposes the same adoption constraint in expectation. The interesting finding is to see that the robust policy is also pricier relative to the flexible setting. In addition, this extra spending does not allow any reduction in the variance of the sales, as was the case when comparing the committed and flexible settings. This analysis supports the fact that the robust policy is not a very desirable option for policy makers, as it is somewhat dominated by the flexible policy in terms of the tradeoff between expected spending and variance of sales.

2. *Semiflexible Policy.* We now consider the second approach, that is, relaxing the target adoption constraint under the flexible policy—i.e., requiring the target to be met under the expectation of both noises.

Figure 2. Sequence of Events Under the Semiflexible Setting



This allows us to consider an intermediate policy that can be compared to the committed setting. Note that in this case, the decision about the second-period subsidy should be made before the realization of the first noise. In contrast with the committed policy, the government can choose an adjustable policy $r_2(s_1)$ that will be contingent on the sales realization. In other words, the decision of r_2 is now adaptive and depends on the realization of the first-period sales s_1. The timing of decisions is depicted in Figure 2. This setting is still flexible as the government can adapt the subsidy level depending on the sales realization. In addition, since the government decides r_2 before the realization of ϵ_1, it allows us to consider an adoption target on expectation with respect to both noises.

As a result, the semiflexible policy imposes the same adoption constraint as the committed setting. It is not a fully flexible policy because the government cannot update the policy after the realization of the demand. As before, we compare the semiflexible policy to both the committed and flexible cases. In particular, we extend all of the results of the paper (i.e., subsidy levels, expected spending, and variance of the sales) for the semiflexible policy. For simplicity, we assume that there is no correlation factor between the time periods—i.e., $\alpha = 0$.

Consider the three settings: committed, flexible, and semiflexible (denoted by the superscripts c, f, and s, respectively). One can show that for any n point discrete distribution of the noise ϵ_1, the optimal subsidy level $r_2(s_1)$ does not depend explicitly on s_1 but only on $E(s_1)$. We also show that

$$r_1^s = r_1^f \quad \text{and} \quad r_2^s = E[r_2^f].$$

In addition, we show that the production decisions are similar to the flexible setting:

$$u_1^s = u_1^f \quad \text{and} \quad u_2^s = E[u_2^f].$$

Note that in the semiflexible policy, both decisions r_2^s and u_2^s are deterministic, whereas in the flexible setting, these decisions are random variables. We next compare the expected spending and the variance of the sales in the three different settings.

Proposition 6. *Consider the three settings: committed, flexible, and semiflexible (denoted by the superscripts c, f, and s, respectively). Then, we have the following.*

- *The expected government spending satisfies relation:* $E[Spending^c] \le E[Spending^s] \le E[Spending^f]$.
- *The variance of the sales follows:* $\mathrm{Var}(s^f) \le \mathrm{Var}(s^s) \le \mathrm{Var}(s^c)$.

Even though the decision variables in the flexible and semiflexible settings are the same in expectation, the expected spending is different. Interestingly, the expected spending under the semiflexible policy happens to lie between the expected spending in the committed and flexible settings. Recall that one of the key messages of the paper is that the flexible policy is more costly relative to the committed setting in terms of expected spending. At the same time, the flexible policy allows one to reduce the variance of the sales, and hence provides a higher confidence in the target adoption. In Proposition 6, we extend the results of the paper for the semiflexible policy that considers the same constraint as the committed setting, where the expectation is taken over both noises. Since this setting considers less information (we do not know the realization of the first-period sales), the semiflexible policy will be less costly relative to the flexible setting. The interesting finding relates to the fact that the semiflexible setting remains pricier relative to the committed setting. Therefore, one can see that the main message of our paper is preserved when we consider the same constraint in both settings. More precisely, the key property is not the difference in adoption constraints, but rather the order of events and the fact that the government does not know the sales realization before setting the subsidy level. Namely, the flexible nature of the policy (i.e., the government can revise the subsidy level depending on the production decision u_1) reduces the variability effect (r_2 becomes now deterministic), but still induces the undersupply incentive of the supplier, and hence incurs higher expected spending relative to the committed case. In addition, this extra spending allows a reduction in the variance of the sales, so that the government acquires a higher confidence about the adoption level.

4.5. Incorporating a Discount Factor

One can extend the results of this paper for the model with a discount factor $0 < \beta < 1$. In particular, Theorem 1 still holds—i.e., the expected spending is larger in the flexible setting, assuming that the correlation factor α is not too negative. The exact threshold of the value of α will now depend on the value of the discount factor. Alternatively, one can consider that the correlation factor α is given, and show that there exists a threshold value on the discount factor (that depends on α), under which our result holds. Mathematically, for any given α, there exists a threshold value on the discount factor, $\beta^-(\alpha)$, such that $E[Sp^f - Sp^c] \ge 0$, $\forall\, 0 < \beta < \beta^-(\alpha)$.

The intuition of this result goes as follows. First, observe that the problem solved by the supplier can easily be modified to account for a discount factor $0 < \beta < 1$ by shifting the price and the cost of the second time period. More precisely, by simply changing

p_2 to βp_2 and c_2 to βc_2, we can apply the exact same methodology. Regarding the optimization problem of the government, things become less straightforward. By repeating the full derivation, one can extend all of our results. In particular, one can show that for $\alpha \geq 0$, the spending gap is positive, and as β decreases, the spending gap becomes smaller. By discounting the government spending in the second period, it becomes more attractive for the government to defer some of the subsidy to the second period, hence making the gap between committed and flexible smaller. Qualitatively, all of the results and insights of this paper extend for any discount factor $0 < \beta < 1$.

5. Computational Experiments

In this section, we develop a numerical experiment to illustrate the impact of varying profit margins and demand uncertainty on the government spending and the supplier's profit level. The numbers used for these simulations are based on the German solar photovoltaic market. Further details on the calculations used to develop the computational experiments can be found in Section A.10 of the appendix. In summary, the data input for this simulation consists of the government adoption target and the basic market parameters such as price, cost, average nominal demand (in the absence of rebate), demand sensitivity to rebates, and salvage value. Some of the parameters used are based on historical figures, while others are roughly estimated (such as the demand sensitivity). To demonstrate the effects of market conditions on committed/flexible policies, the second-period costs and variance of the demand uncertainty are chosen at various levels. The demand uncertainty is drawn from a uniform distribution. Experiments with other distributions have yielded the same qualitative results; therefore, they will not be displayed. Finally, we vary the degree of demand correlation across time periods to illustrate its impact on expected spending. It should be made clear that the data used in this section are used only as a basis for the simulation, which is meant to develop intuition about our model and is not an empirical investigation.

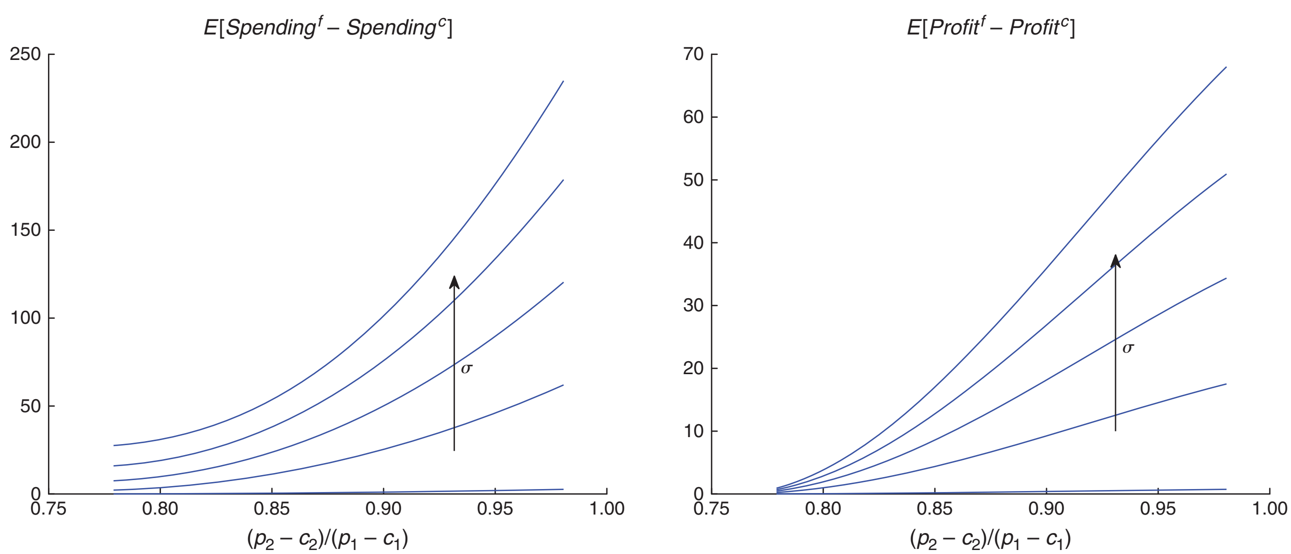
In the simulations presented in Figures 3 and 4, spending and profits are displayed in millions of euro. Sales are measured in megawatts of installed solar panels. The adoption target used to base this simulation was the 7,500 MW sold in Germany during the year 2011. For Figures 3 and 4, we assume there is no intertemporal correlation in demand, $\alpha = 0$. Correlation is introduced later in Figure 5.

In Figure 3, we observe the difference between the flexible and the committed settings in expected government spending and supplier's profit. The horizontal axis displays the level of the firm's second-period profit margin, relative to the first. The vertical axis is the difference in expected spending (left graph) and expected profit (right graph). A few observations are in order.

Observation 1. Between the two settings, the difference in expected spending, as well as supplier profit, converges to zero when demand uncertainty decreases, $\sigma \to 0$. This is to be expected, since the three effects displayed in Theorem 1 disappear without demand uncertainty. In the presence of uncertainty, *supply incentive effect* and the *adaptability effect* drive the difference in spending.

Observation 2. When the profit margin of the second period is much smaller than the first, $(p_2 - c_2)/(p_1 - c_1) \to 0$, the difference in profit for the supplier between

Figure 3. (Color online) Expected Spending and Profits When Varying the Ratio of Profit Margins



the two settings also goes to zero. This occurs because the *supply incentive effect* disappears. This can be largely explained by the convergence of the ordering quantiles k_1^c and k_1^f. When the second-period sales are not very profitable, the underage cost of the supplier in the flexible case is not effectively mitigated by increased demand in the second period. With less incentive to undersupply from the industry side, the amount of subsidies needed from the government gets closer to the committed case. Note that the *correlation effect* is also absent in this example, since $\alpha = 0$. The remaining difference in spending converges to the remaining *adaptability effect*: $\mathrm{VAR}(\min\{k_1^f, w_1\})/b_2$. This term decreases with the magnitude of the demand uncertainty, or equivalently the standard deviation σ. When the profit margin ratio is 0.8, the difference is primarily driven by the adaptability effect. However, when the profit margin ratio is 0.95, the supplier incentive effect becomes three times larger relative to the adaptability effect.

Observation 3. For the simulation, we assume that there is a baseline feed-in-tariff of 0.25€/kWh (based on residential electricity prices) that would lead to the nominal demand levels. To reach the desired 7,500 MW of installations, we introduce additional subsidies that cost the government somewhere between 870 and 915 million euro in a committed setting (depending on profit margins and demand uncertainty). As shown in Figure 3, the expected spending in a flexible setting can be as high as 235 million euro more than the committed spending, when demand uncertainty is large and the profit margin of the second period is close to the profit of the first period. In other words, the additional flexibility premium is close to 25% of the cost of the subsidy program under policy commitment. This indicates that commitment versus flexibility should be a significant concern for policy makers. It is important to reiterate that the numbers presented here are only used to show the potential impact of a flexible/committed policy and are not meant to be used to evaluate past policy decisions.

In Figure 4, we observe the difference in the standard deviation of sales and government spending between the flexible and the committed settings. As before, the horizontal axis displays the level of the firm's second-period profit margin, relative to the first. The vertical axis is the difference in the standard deviation of sales, measured in megawatts of installed solar panels (left graph) and the standard deviation of spending measured in millions of euro (right graph).

Observation 4. The variance of the total sales is indeed smaller, as expected from Theorem 2. The higher expected spending is indeed lowering the variance of the sales in the flexible setting, allowing the government to be closer to the adoption target. On the other hand, the variance of spending is not necessarily lower in the flexible setting. In fact, when the profit margin of the second period is high enough, Figure 4 shows that the standard deviation in spending is lower in the flexible case. This is driven by the low variance of the sales in the first period.

When the profit margin of the second period is too low, the standard deviation in spending is actually higher in the flexible case. Without the undersupply incentive, the sales in the first period of both the flexible and committed settings converge. Therefore, both settings have variable sales in the first period. In the second period, the variance of committed spending is mostly determined by the underlying demand

Figure 4. (Color online) Standard Deviation of the Sales and the Spending When Varying the Ratio of Profit Margins

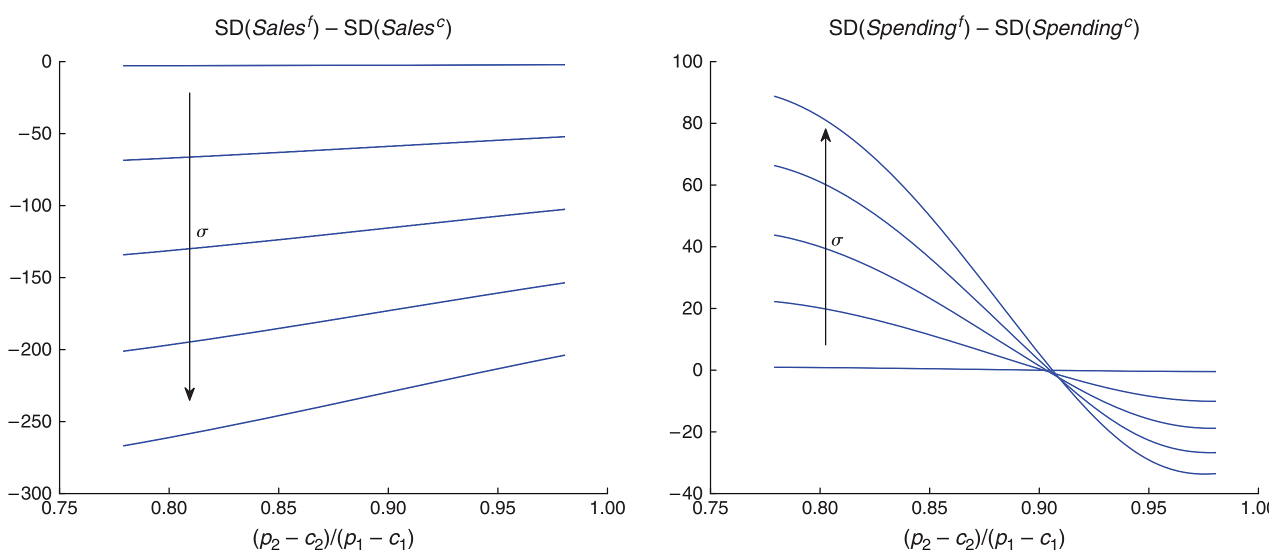


Figure 5. (Color online) Expected Spending When Varying α and σ

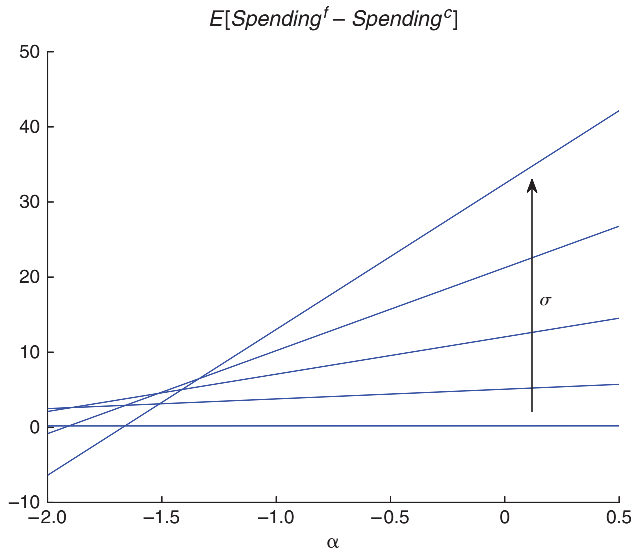
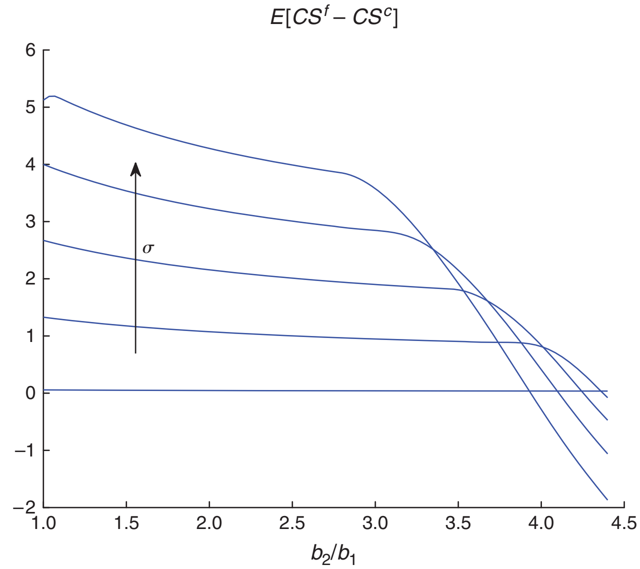


Figure 6. (Color online) Expected Consumer Surplus When Varying the Ratio of Price Sensitivities



uncertainty. In the flexile setting, the policy readjustment compounds the variance of the first-period sales with the second period. This increases the variance in the spending distribution.

In Figure 5, we present the effect of demand correlation on the expected government spending. For $\alpha \geq 0$, the relationship of Corollary 1 is verified: $E[Spending^f - Spending^c] > 0$. Interestingly, when demand uncertainty, σ, is sufficiently high and the correlation is sufficiently negative (for instance, $\alpha = -2$), the relationship can be inverted: $E[Spending^f - Spending^c] < 0$. This means that the *correlation effect* becomes dominant in Theorem 1. As seen in Theorem 1, the average spending for the committed setting does not depend on α. The flexible spending is what changes with α, as displayed in Figure 5.

A positive α means that low initial demand is followed by a lower average demand later. In the flexible setting, the government will overcompensate in subsidies to get back close to the adoption target. At the same time, subsidizing becomes increasingly expensive when arriving in the second period with low sales. When there is high early demand, the flexible government can reduce spending in the second period, but the benefits of high early demand are curbed by the limited supply.

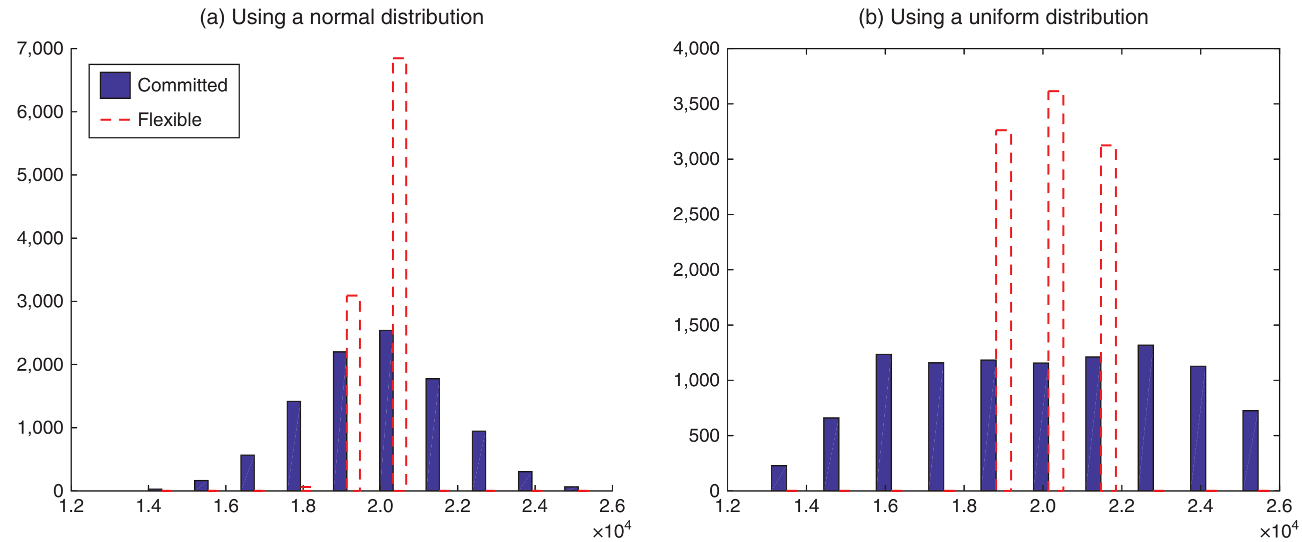
With negative correlation, low initial demand is compensated by high demand later. High early demand leads to lower demand later. This effectively works as a natural hedge for the flexible government and can outweigh the other effects described in Theorem 1.

We next show computationally that the total expected consumer surplus can be larger or smaller in the flexible setting depending on the price sensitivity ratio. In Figure 6 is shown the ratio $E[CS^c]/E[CS^f]$ as a function of the ratio of price sensitivities b_2/b_1. More specifically, in this experiment, we fixed b_1 at the original estimated value and varied only b_2.

Observation 5. One can see that the total expected consumer surplus inequality can go either way depending on the value of b_2/b_1. When b_2/b_1 is not very large, the expected consumer surplus is higher in the flexible setting. However, for large values of b_2/b_1, the situation is reverted. This region, where $E[CS^f - CS^c] < 0$, represents the regime where the benefit of higher subsidies in the flexible setting are dominated by the increased risk of early stockouts. As shown in Theorem 4, the flexible setting does not always benefit consumers in terms of expected consumers surplus. We further note that the threshold value of b_2/b_1 is decreasing in the magnitude of the demand uncertainty.

In Theorem 2, we show that the variance of the sales is lower in the flexible setting. Recall that both policies have the same expected adoption level. We next compare the histograms of adoption levels in both settings. In particular, we are interested in the scenarios where the total realized sales exceed the target level Γ. We consider several settings with different parameters and noise distributions, and observe consistently that the probability of the final adoption is higher in the flexible setting. In Figure 7, we present the results when Γ = 20,000 and the correlation parameter $\alpha = 3$, by drawing 10,000 independent random samples of the noise (similar results can be obtained when $\alpha = 0$). In Figure 7(a), we consider a (truncated) normally distributed noise with $N[\mu_1, \sigma = 800]$, whereas in Figure 7(b), we use a uniform noise $U[-0.9\mu_1, 0.9\mu_1]$. Here, μ_1 corresponds to the value of the nominal demand in the first period.

Figure 7. (Color online) Histogram of Adoption Levels in Both Settings



As we can see from Figure 7(a), for the normal noise, the flexible policy attains the target adoption in 80% of the cases, versus 46% in the committed setting. From Figure 7(b), when the noise is uniform, the flexible policy attains the target adoption in 68% of the cases, versus 47% in the committed. This test allows us to strengthen the main message of this paper: The flexible policy is more expensive but allows the government to achieve a higher probability of reaching the target adoption. More precisely, the variance of the sales is lower, and the final adoption is more likely to exceed the target.

We next present some additional insights on the welfare implications. More precisely, we are interested in comparing the value of the expected welfare under both regimes. We assume that the government's objective is to maximize the total welfare from Equation (10) with a quadratic externality function:

$$Externality = \theta\, Sales - \lambda\, Sales^2.$$

The parameter θ is set to a large value to ensure that the total welfare is positive. In particular, we take $\theta = 10^6$ and vary the value of λ. In Figure 8(a), we vary λ between 0 and 100, whereas in Figure 8(b), we vary λ between 0 and 1,000. In each case, we plot the relative difference in expected welfare between the flexible and committed policies. As expected, when $\lambda = 0$, both settings coincide and yield the same welfare value. When $\lambda > 0$, one can see that the welfare under the flexible policy is higher relative to the committed setting. For example, when $\lambda = 100$, we obtain a 4.8% relative welfare increase. This suggests that even though the

Figure 8. (Color online) Relative Difference in Expected Welfare (with $\theta = 10^6$)

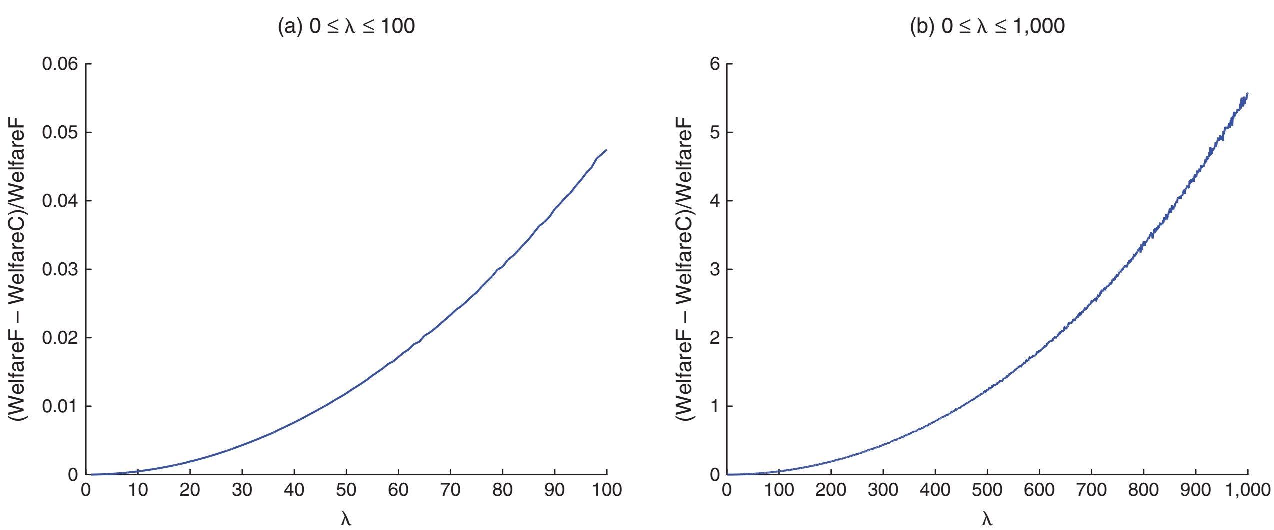
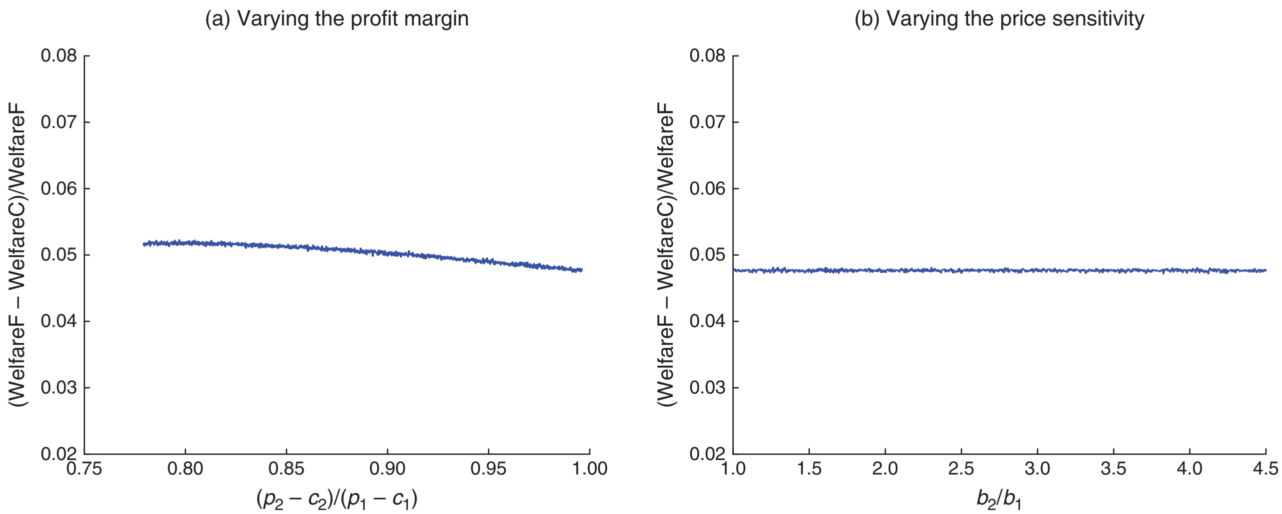


Figure 9. (Color online) Relative Difference in Expected Welfare (with $\theta = 10^6$ and $\lambda = 100$)



committed policy yields lower expected spending, it also leads to a lower expected welfare. Consequently, the comparison between the two regimes is more subtle and will depend on the specific welfare function and the relative importance to the government of low spending and high welfare.

In Figure 9, we further investigate the relative welfare difference as a function of the level of the firm's second-period profit margin relative to the first (Figure 9(a)), and as a function of the ratio of price sensitivities b_2/b_1 (Figure 9(b)). Interestingly, we can see that the relative difference in expected welfare is stable and is not affected by variations in these parameters. This confirms the fact that the expected welfare is higher under the flexible policy when using such a quadratic externality function.

6. Conclusions

Flexibility can be seen as an asset in many operations management applications. When the government is designing consumer subsidies, policy flexibility can clearly be a liability. This result comes from the fact that industry is strategically responding to the policy design. Under a flexible policy, the firm will supply less in the early stage, relative to a committed setting. This is due to the fact that a low demand in the earlier period can be compensated by the government in the future, creating an undersupply incentive for the firm. This ultimately increases the total cost of the subsidy program.

This result carries a potentially significant qualitative insight for policy makers. The constant readjustment of the subsidy policies can cause serious adverse effects in the production incentives. While there is evidence that a flexible policy is being implemented in Germany, it is hard to obtain a counterfactual policy where the feed-in-tariff levels were precommitted over the years. We show in this paper that by implementing the flexible policy, the German government might be inducing an undersupply in the industry that makes the policy on average more expensive, but that reduces the variance in the final level of adoption. This theoretical result provides a practical insight for future policy makers: the frequency of their policy revision carries an important trade-off between expected spending and the uncertainty about how many customers will adopt the green product. Flexible policies are more expensive, but reduce uncertainty about the adoption level (and may also increase the total expected welfare). As a result, it is not clear what the preferred policy is from the government perspective. Depending on the relative importance of the expected spending (i.e., the budget), the expected total welfare, and the variance of the sales (i.e., the likelihood of reaching a target adoption), the government may decide to adopt the committed or the flexible policy. Consequently, governments with the lack of ability to commit will suffer a higher expected spending, but may attain a higher expected welfare.

We also have shown that a significant negative demand correlation across time periods creates an advantage for the flexible policy. Under negative correlation, the flexible spending might even be smaller relative to the committed spending. It is interesting to note that acquiring new demand information is not universally better for the flexible policy. In fact, only negative demand correlation provides a benefit to flexible policies in terms of average spending.

Finally, we note that on average, because of the reduced cost of undersupplying, firms benefit from flexible subsidy policies. Consumers may be better off or worse off with respect to policy commitment. Flexibility creates lower initial supply levels, which translates into higher stockout risk. At the same time, it increases the average subsidy level. From a consumer's perspective, the trade-off between higher subsidy and higher stockout probability will depend on the relative price elasticity of early customers and late customers.

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Appendix

A.1. Proof of Lemma 1

We first consider the committed setting. We denote by $h_2^c(x_2)$ and $H_2^c(x_2, u_2)$ the second-period supplier's profit-to-go and objective, respectively. Consider the supplier's problem at $t = 2$:

$$\begin{aligned} h_2^c(x_2) &= \max_{u_2 \ge 0} H_2^c(x_2, u_2) \\ &= \max_{u_2 \ge 0} \{p_2 E[\min(x_2 + u_2, b_2 r_2 + \mu_2 + \alpha w_1 + w_2)] - c_2 u_2 \\ &\quad + p_3 E[\max(x_2 + u_2 - b_2 r_2 - \mu_2 - \alpha w_1 - w_2, 0)]\}. \end{aligned} \tag{A.1}$$

Let $\hat{u}_2$ be the solution of the first-order condition of the problem above, which follows

$$p_2 P(x_2 + \hat{u}_2 \le b_2 r_2 + \mu_2 + \alpha w_1 + w_2) - c_2 + p_3 P(x_2 + \hat{u}_2 \ge b_2 r_2 + \mu_2 + \alpha w_1 + w_2) = 0,$$

which is equivalent to

$$p_2(1 - F_{w_2}(x_2 + \hat{u}_2 - b_2 r_2 - \mu_2 - \alpha w_1) - c_2 + p_3 F_{w_2}(x_2 + \hat{u}_2 - b_2 r_2 - \mu_2 - \alpha w_1) = 0.$$

The unique solution to the first-order condition is given by

$$\begin{aligned} \hat{u}_2 &= b_2 r_2 - x_2 + \mu_2 + \alpha w_1 + F_{w_2}^{-1}\left(\frac{p_2 - c_2}{p_2 - p_3}\right) \\ &= b_2 r_2 - x_2 + \mu_2 + \alpha w_1 + k_2^c. \end{aligned}$$

In addition, the second derivative of the objective function is nonpositive:

$$\frac{d^2 H_2^c(x_2, u_2)}{du_2^2} = -p_2 f_{\epsilon_2}(x_2 + u_2 - b_2 r_2) + p_3 f_{\epsilon_2}(x_2 + u_2 - b_2 r_2) \le 0.$$

Here, $f_{\epsilon_2}(\cdot)$ is the probability density function (pdf) of ϵ_2 (remember that $\epsilon_2 = w_2 + \mu_2 + \alpha w_1$, which is always positive). Since $p_2 > p_3$, the second-order condition is satisfied and $\hat{u}_2$ is the maximizer of the unconstrained problem.

From Assumption 2, we have $c_2 > p_3$. If that was not the case, the supplier could produce an infinite number of units during the second period at a cost below the salvage value, making infinite profits. Since $c_2 > p_3$, in the limit $u_2 \to \infty$, the objective function goes to $H_2^c(x_2, u_2) \to -\infty$. From continuity of $H_2^c(x_2, u_2)$, there is a solution of the maximization problem above, which must be either at the boundary $u_2 = 0$ or satisfying the first-order condition—i.e., $u_2 = \hat{u}_2$.

Since the objective value is finite at $u_2 = 0$ and $-\infty$ when $u_2 \to \infty$, the objective function $H_2^c(x_2, u_2)$ is nonincreasing with respect to u_2 for any $u_2 \ge \hat{u}_2$. Therefore, the optimal second-period ordering level in the committed setting is given by $u_2^*(x_2, r_2) = \max(b_2 r_2 - x_2 + \mu_2 + \alpha w_1 + k_2^c, 0)$. At the first period, the manufacturer is maximizing the expected first-period profit plus the profit-to-go of the second period:

$$\begin{aligned} &\max_{u_1 \ge 0} \{p_1 E[\min(x_1 + u_1, b_1 r_1 + \epsilon_1)] - c_1 u_1 \\ &\quad + E[h_2^c(x_2(x_1, u_1, r_1, \epsilon_1))]\}. \end{aligned} \tag{A.2}$$

We define the following first-period production quantity: $\hat{u}_1 = b_1 r_1 - x_1 + k_1^c + \mu_1$.

We next show that this quantity satisfies the first-order condition of problem (A.2). First, note that under this policy and Assumption 3, we obtain the no-idling condition. If there is any leftover inventory, it will be given by

$$\begin{aligned} x_2 &= x_1 + \hat{u}_1 - b_1 r_1 - \mu_1 - w_1 = k_1^c - w_1 \\ &\le k_2^c + \mu_2 + \alpha w_1 \le b_2 r_2 + k_2^c + \mu_2 + \alpha w_1. \end{aligned}$$

The first inequality follows from Assumption 3 and the second from the nonnegativity of the subsidy level. Therefore, the optimal second-period ordering policy simplifies to $u_2^*(x_2, r_2, w_1) = b_2 r_2 - x_2 + k_2^c + \mu_2 + \alpha w_1$, which is nonnegative. Under the optimal ordering policy, the expected profit-to-go is given by

$$\begin{aligned} E[h_2^c(x_2)] &= p_2(b_2 r_2 + \mu_2 + E[\min(k_2^c, w_2)] \\ &\quad - c_2(b_2 r_2 - x_2 + \mu_2 + k_2^c) + p_3 E[\max(k_2^c - w_2, 0)], \end{aligned}$$

where we used $E[w_1] = 0$. We next compute the derivative of the expected profit-to-go function:

$$\frac{dE[h_2^c]}{du_1} = E\left[\left(\frac{dh_2^c}{dx_2}\right)\left(\frac{dx_2}{du_1}\right)\right] = (c_2)(F_{w_1}(x_1 + u_1 - b_1 r_1 - \mu_1)).$$

Therefore, the first-order condition of equation (A.2) can be expressed as

$$\begin{aligned} &p_1(1 - F_{w_1}(x_1 + u_1 - b_1 r_1 - \mu_1)) \\ &\quad - c_1 + c_2 F_{w_1}(x_1 + u_1 - b_1 r_1 - \mu_1) = 0. \end{aligned}$$

Note that $\hat{u}_1$ is the unique solution to the expression above. Note also that the second-order derivative is negative, guaranteeing optimality: $-p_1 f_{\epsilon_1}(x_1 + u_1 - b_1 r_1)) + c_2 f_{\epsilon_1}(x_1 + u_1 - b_1 r_1) < 0$. This follows from the facts that $p_1 > p_2 > c_2$ and the pdf $f_{\epsilon_1}(\cdot)$ is always positive. Therefore, the optimal solution is either at $u_1 = \hat{u}_1$ or at the boundary—i.e., $u_1 = 0$. From Assumption 3, we know that $\hat{u}_1 > 0$, and as a result, $u_1^*(x_1, r_1) = b_1 r_1 - x_1 + k_1^c + \mu_1$.

We next consider the government problem in the committed setting:

$$\begin{aligned} \min_{r_1, r_2 \ge 0} \quad & \{r_1 E[s_1(x_1, u_1^{c*}(x_1, r_1), r_1, \epsilon_1)] \\ & + r_2 E[s_2(x_2, u_2^{c*}(x_2, r_2), r_2, \epsilon_2)]\} \\ \text{s.t.} \quad & E[s_1(x_1, u_1^{c*}(x_1, r_1), r_1, \epsilon_1)] \\ & + E[s_2(x_2, u_2^{c*}(x_2, r_2), r_2, \epsilon_2)] \ge \Gamma, \\ & \text{where } s_t(x_t, u_t, r_t, \epsilon_t) = \min(x_t + u_t, b_t r_t + \epsilon_t) \\ & \qquad x_{t+1} = x_t + u_t - s_t(x_t, u_t, r_t, \epsilon_t). \end{aligned} \tag{A.3}$$

Using the optimal production quantities, $u_1^{c*}(x_1, r_1)$ and $u_2^{c*}(x_2, r_2)$, derived above, we obtain the expected sales levels: $E[s_t(x_t, u_t, r_t, \epsilon_t)] = b_t r_t + \mu_t + E[\min(k_t^c, w_t)] = b_t r_t + \mu_t + v_t^c$. As a result, the optimization problem reduces to

$$\begin{aligned} \min_{r_1, r_2 \geq 0} \quad & \{r_1(b_1 r_1 + \mu_1 + v_1^c) + r_2(b_2 r_2 + \mu_2 + v_2^c)\} \\ \text{s.t.} \quad & (b_1 r_1 + \mu_1 + v_1^c) + (b_2 r_2 + \mu_2 + v_2^c) \geq \Gamma. \end{aligned} \tag{A.4}$$

The objective function is nondecreasing in both r_1 and r_2, and the expected sales is a continuous function. Therefore, the optimal solution must occur when the adoption constraint is exactly met. We can solve this by expressing r_1 as a function of r_2: $r_1 = (\Gamma - v_1^c - b_2 r_2 - v_2^c - \mu_1 - \mu_2)/b_1$. The problem becomes

$$\begin{aligned} \min_{r_2 \geq 0} \Bigg\{ & \frac{\Gamma - v_1^c - b_2 r_2 - v_2^c - \mu_1 - \mu_2}{b_1} (\Gamma - b_2 r_2 - v_2^c - \mu_2) \\ & + r_2(b_2 r_2 + \mu_2 + v_2^c) \Bigg\}. \end{aligned} \tag{A.5}$$

Note that the objective function is convex in r_2. By taking the first-order condition, we obtain

$$r_2^{c*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_2^c + \mu_2)(b_1 + 2b_2)}{2b_2(b_1 + b_2)} - \frac{v_1^c + \mu_1}{2(b_1 + b_2)}.$$

The first-period subsidy value follows from the target constraint and is given by

$$r_1^{c*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^c + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2^c + \mu_2}{2(b_1 + b_2)}.$$

Similarly, we next solve the flexible problem by starting from the second period. The derivation of the second-period production level u_2 is the same as in the committed setting. In particular, the problem can be written as

$$\begin{aligned} h_2^f(x_2, r_2) = \max_{u_2 \geq 0} \{ & p_2 E[\min(x_2 + u_2, b_2 r_2 + \mu_2 + \alpha w_1 + w_2)] - c_2 u_2 \\ & + p_3 E[\max(x_2 + u_2 - b_2 r_2 - \mu_2 - \alpha w_1 - w_2, 0)]\}. \end{aligned} \tag{A.6}$$

The optimal ordering quantity can be expressed as $u_2^*(x_2, r_2) = \max(b_2 r_2 - x_2 + k_2^f + \mu_2 + \alpha w_1, 0)$. The government optimization problem at the second period is given by

$$\begin{aligned} g(s_1, x_2) = \min_{r_2} \; & r_2 E[s_2(x_2, u_2^*(x_2, r_2, w_1), r_2, w_1)] \\ \text{s.t.} \quad & s_1 + E[s_2(x_2, u_2^{c*}(x_2, r_2, w_1), r_2, w_1)] \geq \Gamma. \end{aligned} \tag{A.7}$$

By using the optimal ordering quantity, we obtain $E[s_2(x_2, u_2, r_2, \epsilon_2)] = b_2 r_2 + \mu_2 + v_2^f$. One can see that both the objective function and the adoption constraint are nondecreasing with respect to r_2. Therefore, the optimal solution can be obtained when the adoption constraint is exactly met:

$$r_2^{f*}(s_1, x_2) = \frac{\Gamma - s_1 - v_2^f - \mu_2 - \alpha w_1}{b_2}.$$

We next solve the problem faced by the supplier at the first period:

$$\begin{aligned} \max_{u_1 \geq 0} \{ & p_1 E[s_1(x_1, u_1, r_1, w_1)] - c_1 u_1 \\ & + E[h_2^f(x_2(x_1, u_1, r_1, w_1), r_2^*(s_1(x_1, u_1, r_1, w_1), \\ & \quad x_2(x_1, u_1, r_1, w_1)))]\}. \end{aligned} \tag{A.8}$$

As in the committed setting, we assume that the manufacturer does not idle in the second period. Note that we have $h_2^f = p_2(b_2 r_2 + \mu_2 + v_2^f + \alpha w_1) - c_2(b_2 r_2 + \mu_2 + \alpha w_1 - x_2 + k_2^f) + p_3 E[\max(k_2^f - w_2, 0)]$. Substituting the second-period subsidy level, we obtain $h_2^f = p_2(\Gamma - s_1) - c_2(\Gamma - s_1 - x_2 + k_2 - v_2) + p_3 E[\max(k_2^f - w_2, 0)]$. Note also that $s_1 = \min(x_1 + u_1, b_1 r_1 + \mu_1 + w_1)$ and $dE[h_2^f]/du_1 = -p_2(1 - F_{\epsilon_1}(x_1 + u_1 - b_1 r_1)) + c_2$. The first-order condition on problem (A.8) yields

$$\begin{aligned} & p_1(1 - F_{\epsilon_1}(x_1 + u_1 - b_1 r_1)) - c_1 \\ & \quad - p_2(1 - F_{\epsilon_1}(x_1 + u_1 - b_1 r_1)) + c_2 = 0. \end{aligned}$$

Equivalently, $(p_1 - p_2)F_{\epsilon_1}(x_1 + u_1 - b_1 r_1) = p_1 - c_1 - p_2 + c_2$. Note that the second derivative is negative, since $p_1 > p_2$, which implies that the objective function is concave. One see that the following first-period production quantity uniquely satisfies the first-order condition written above: $\hat{u}_1^f = b_1 r_1 - x_1 + k_1^f + \mu_1$. Since from Assumption 3 we have $x_1 \leq k_1^f + \mu_1$, we know that the optimal solution is positive and therefore $u_1^{f*} = \hat{u}_1^f$.

Under this policy, we have $s_1(x_1, \hat{u}_1, r_1, w_1) = b_1 r_1 + \mu_1 + \min(k_1^f, w_1)$. In addition, $x_2 = x_1 + u_1 - s_1 = k_1^f - \min(k_1^f, w_1)$. Therefore, the second-period subsidy level can be expressed as the first-period subsidy as follows: $r_2^{f*}(s_1, x_2) = (\Gamma - b_1 r_1 - \mu_1 - \min(k_1^f, w_1) - \mu_2 - \alpha w_1 - v_2^f)/b_2$.

The first-period government problem is given by

$$\min_{r_1} r_1 E[s_1(x_1, u_1^*(r_1), r_1, w_1)] + E[g(s_1(x_1, u_1^*(r_1), r_1, w_1))]. \tag{A.9}$$

From the solution of the second-period problem, we have

$$\begin{aligned} g(s_1) &= \frac{\Gamma - s_1 - v_2^f}{b_2}(\Gamma - s_1) \\ &= \frac{\Gamma - (b_1 r_1 + \min(k_1^f, \epsilon_1)) - v_2^f}{b_2}[\Gamma - (b_1 r_1 + \min(k_1^f, \epsilon_1))]. \end{aligned}$$

The optimal subsidy level for the first period can then be obtained by solving the first-order condition of problem (A.9). We further note that the second derivative is always positive, indicating that the function is convex. This solution is given by

$$r_1^{f*} = \frac{\Gamma}{b_1 + b_2} - \frac{(v_1^f + \mu_1)(2b_1 + b_2)}{2b_1(b_1 + b_2)} - \frac{v_2^f + \mu_2}{2(b_1 + b_2)}. \quad \square$$

A.2. Proof of Proposition 1

- From the definitions of the ordering quantiles in Table 1, we have $k_1^c = F_{w_1}^{-1}(1 - (c_1 - c_2)/(p_1 - c_2))$ and $k_1^f = F_{w_1}^{-1}(1 - (c_1 - c_2)/(p_1 - p_2))$.

We assume w_1 to be continuously distributed with full support on $[A_1, B_1]$. Since the function $F_{w_1}^{-1}$ is increasing, we only need to show $1 - (c_1 - c_2)/(p_1 - c_2) > 1 - (c_1 - c_2)/(p_1 - p_2)$. This can be implied from $p_1 - c_2 > p_1 - p_2$, which is true from Assumption 2: $p_2 > c_2$. Therefore, $k_1^c > k_1^f$. For the second time period, the relationship is trivially true from the definition $k_2^c = k_2^f$.

Additionally, from the definition of the expected sales quantiles, we have the relations $v_1^c = E[\min(k_1^c, w_1)]$ and $v_1^f = E[\min(k_1^f, w_1)]$. Note that $\min(k_1^c, w_1) \geq \min(k_1^f, w_1)$ for any value of w_1. Since the distribution is fully supported in

$[A_1, B_1]$ and we know that $A_1 < k_1^f < k_1^c < B_1$, there will be some measurable part of the distribution where the inequality is strict: $\min(k_1^c, w_1) > \min(k_1^f, w_1)$. Therefore, we obtain $v_1^c > v_1^f$. In addition, recall that we have $v_2^c = v_2^f$.

- Using the optimal ordering policies and subsidy levels from Lemma 1, the expected sales are

$$E[s_t] = E[\min\{x_t + u_t, b_t r_t + \epsilon_t\}]$$
$$= b_t E[r_t^{j^*}] + \mu_t + E[\min\{k_t^j, w_t\}] = b_t E[r_t^{j^*}] + \mu_t + v_t^j,$$

where j can be either c or f for the committed or flexible setting, respectively. Note that the correlation effect is additive with zero mean, therefore not appearing in the expectation of sales. The first- and second-period expected sales level will be given by

$$E[s_1^j] = \frac{b_1\Gamma}{b_1+b_2} + \frac{b_2(v_1^j+\mu_1)}{2(b_1+b_2)} - \frac{b_1(v_2^j+\mu_2)}{2(b_1+b_2)} \quad \text{and}$$
$$E[s_2^j] = \frac{b_2\Gamma}{b_1+b_2} + \frac{b_1(v_2^j+\mu_2)}{2(b_1+b_2)} - \frac{b_2(v_1^j+\mu_1)}{2(b_1+b_2)}.$$

Note that the average sales maintain the same structure between the two settings. The only difference is the first expected sales quantile v_1^c and v_1^f. We can now calculate the difference in expected sales:

$$E[s_1^c - s_1^f] = \frac{b_2}{2(b_1+b_2)}(v_1^c - v_1^f) > 0 \quad \text{and}$$
$$E[s_2^c - s_2^f] = -\frac{b_2}{2(b_1+b_2)}(v_1^c - v_1^f) < 0.$$

One can also see that $E[s_1^j] + E[s_2^j] = \Gamma$, for both $j \in \{c, f\}$, as expected.

- By using Lemma 1, the expressions for u_1^c and u_1^f are given by

$$u_1^c = b_1 r_1^c + k_1^c + \mu_1 - x_1 \quad \text{and} \quad u_1^f = b_1 r_1^f + k_1^f + \mu_1 - x_1.$$

Therefore, one can compute the difference: $u_1^c - u_1^f = b_1 \cdot (r_1^c - r_1^f) + k_1^c - k_1^f$. Next, we substitute the expressions for r_1^c and r_1^f from Lemma 1, given by

$$r_1^c = \frac{\Gamma}{b_1+b_2} - \frac{(v_1^c+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \frac{v_2^c+\mu_2}{2(b_1+b_2)} \quad \text{and}$$
$$r_1^f = \frac{\Gamma}{b_1+b_2} - \frac{(v_1^f+\mu_1)(2b_1+b_2)}{2b_1(b_1+b_2)} - \frac{v_2^f+\mu_2}{2(b_1+b_2)}.$$

So the difference is equal to: $r_1^c - r_1^f = (v_1^f - v_1^c)((2b_1+b_2)/(2b_1(b_1+b_2)))$. Therefore, we obtain

$$u_1^c - u_1^f = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)}. \tag{A.10}$$

Now, since $0 \le (2b_1+b_2)/(2(b_1+b_2)) \le 1$, it remains to show that $k_1^c - k_1^f \ge v_1^c - v_1^f$.

To show $k_1^c - k_1^f \ge v_1^c - v_1^f$, consider the following difference: $v_1^c - v_1^f = E[\min(k_1^c, w_1) - \min(k_1^f, w_1)]$. We must look at each realization of the following random variable: $\min(k_1^c, w_1) - \min(k_1^f, w_1)$. We divide the analysis into cases depending on the realization of w_1.

—*Case* 1: $w_1 \ge k_1^c$. Since $k_1^c \ge k_1^f$, then $\min(k_1^c, w_1) - \min(k_1^f, w_1) = k_1^c - k_1^f \le k_1^c - k_1^f$.

—*Case* 2: $w_1 \le k_1^f$. Since $k_1^c \ge k_1^f$, then $\min(k_1^c, w_1) - \min(k_1^f, w_1) = w_1 - w_1 = 0 \le k_1^c - k_1^f$.

—*Case* 3: $k_1^f \le w_1 \le k_1^c$. We have $\min(k_1^c, w_1) - \min(k_1^f, w_1) = w_1 - k_1^f \le k_1^c - k_1^f$.

Therefore, in each case, $\min(k_1^c, w_1) - \min(k_1^f, w_1) \le k_1^c - k_1^f$. By taking the expectation, we obtain $v_1^c - v_1^f = E[\min(k_1^c, w_1) - \min(k_1^f, w_1)] \le k_1^c - k_1^f$. Therefore, we conclude $u_1^c \ge u_1^f$.

We next compare the expected production quantities for the second time period. Using Lemma 1, the expressions for u_2^c and u_2^f are given by

$$u_2^c = b_2 r_2^c + k_2^c + \mu_2 + \alpha w_1 - x_2^c \quad \text{and}$$
$$u_2^f = b_2 r_2^f + k_2^f + \mu_2 + \alpha w_1 - x_2^f.$$

Note that the only random variables are r_2^f, x_2^c, w_1 (whose mean is zero) and x_2^f, whereas the remaining terms are deterministic. Therefore, by taking the expectation, we obtain

$$E[u_2^c] = b_2 r_2^c + k_2^c + \mu_2 - E[x_2^c] \quad \text{and}$$
$$E[u_2^f] = b_2 E[r_2^f] + k_2^f + \mu_2 - E[x_2^f].$$

The difference is then given by $E[u_2^f] - E[u_2^c] = b_2(E[r_2^f] - r_2^c) + k_2^f - k_2^c + E[x_2^c] - E[x_2^f]$. Recall that $k_2^f = k_2^c$, and therefore $E[u_2^f] - E[u_2^c] = b_2(E[r_2^f] - r_2^c) + E[x_2^c] - E[x_2^f]$. From Proposition 1, we know $E[r_2^f] \ge r_2^c$; hence, we need to show that $E[x_2^c] \ge E[x_2^f]$. We have

$$E[x_2^c] = x_1 + u_1^c - E[s_1^c] \quad \text{and} \quad E[x_2^f] = x_1 + u_1^f - E[s_1^f].$$

So the difference is equal to $E[x_2^c] - E[x_2^f] = u_1^c - u_1^f + E[s_1^f] - E[s_1^c]$. Therefore, we obtain

$$E[x_2^c] - E[x_2^f] = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)} - (v_1^c - v_1^f)\frac{b_2}{2(b_1+b_2)}.$$

By canceling terms, we obtain $E[x_2^c] - E[x_2^f] = k_1^c - k_1^f - (v_1^c - v_1^f)$. By using $k_1^c - k_1^f \ge v_1^c - v_1^f$, we conclude $E[x_2^c] \ge E[x_2^f]$; therefore, $E[u_2^f] \ge E[u_2^c]$.

Finally, we compare the total expected production quantities. The difference between the expected total production quantities is given by

$$u_1^c - u_1^f - E[u_2^f - u_2^c] = k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)}$$
$$- b_2(E[r_2^f] - r_2^c) - (k_1^c - k_1^f) - (v_1^f - v_1^c).$$

By canceling terms, we obtain $u_1^c - u_1^f - E[u_2^f - u_2^c] = (v_1^c - v_1^f) \cdot b_2/(2(b_1+b_2)) - b_2(E[r_2^f] - r_2^c)$. From Lemma 1,

$$u_1^c - u_1^f - E[u_2^f - u_2^c] = (v_1^c - v_1^f)\frac{b_2}{2(b_1+b_2)} - b_2\frac{v_1^c - v_1^f}{2(b_1+b_2)} = 0.$$

Therefore, we conclude $u_1^c + E[u_2^c] = u_1^f + E[u_2^f]$.

- From the definitions of the optimal subsidy levels in Lemma 1, we obtain the difference between the first-period subsidy in the flexible and committed settings:

$$r_1^{c^*} - r_1^{f^*} = -\frac{2b_1+b_2}{2b_1(b_1+b_2)}[v_1^c - v_1^f] < 0.$$

We know that $v_1^c > v_1^f$. Therefore, the subsidy level in the committed setting is smaller: $r_1^{c*} - r_1^{f*} < 0$. For the second-period subsidy, we calculate the expected difference in subsidies:

$$r_2^{c*} - E[r_2^{f*}(s_1)] = -\frac{v_1^c}{2(b_1+b_2)} + \frac{v_1^f}{2(b_1+b_2)} < 0,$$

which is also negative since $v_1^c > v_1^f$. □

A.3. Proof of Theorem 1 and Corollary 1

Under the committed setting, the expected spending levels for each period is easily obtained since the subsidy levels are deterministic. $E[Spending^c] = E[s_1]r_1^{c*} + E[s_2]r_2^{c*}$. The expected sales are given by $E[s_t] = \min\{x_t + u_t, b_t r_t + \epsilon_t\}$. Under the optimal ordering policy from Lemma 1 and considering that $E[w_1] = 0$, we obtain $E[s_t] = b_t r_t^{c*} + \mu_t + E[\min\{k_t^c, w_t\}] = b_t r_t^{c*} + \mu_t + v_t^c$, proving the first relationship. Under a flexible setting, we obtain $E[s_1]r_1^{f*} = b_1 r_1^{f*} + \mu_1 + v_1^f$ in a similar way for the first time period. For the second period, note that both subsidy and sales are random variables. Therefore, using $s_2 = b_2 r_2^{f*} + \mu_2 + \alpha w_1 + \min\{k_2^f, w_2\}$, we obtain

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2 + r_2^f(\mu_2 + \alpha w_1) + \min(k_2^f, w_2) r_2^f] \\
&= E[b_2(r_2^f)^2] + E\left[(\mu_2+\alpha w_1)\frac{\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f}{b_2}\right] \\
&\quad + E\left[\min(k_2^f, w_2)\frac{\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f}{b_2}\right] \\
&= E[b_2(r_2^f)^2] + E\left[(\mu_2+\alpha w_1)\frac{-s_1^f - \alpha w_1}{b_2}\right] \\
&\quad + E\left[(\mu_2+\alpha w_1)\frac{\Gamma - \mu_2 - v_2^f}{b_2}\right] + v_2^f\frac{\Gamma - \mu_2 - v_2^f}{b_2} \\
&\quad + E\left[\min(k_2^f, w_2)\frac{-s_1^f - \alpha w_1}{b_2}\right] \\
&= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - \frac{\alpha\mu_2}{b_2}E[(w_1)] \\
&\quad - E\left[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}\right] + \frac{\mu_2(\Gamma - \mu_2 - v_2^f)}{b_2} \\
&\quad + \frac{\alpha(\Gamma - \mu_2 - v_2^f)}{b_2}E[w_1] + v_2^f\frac{\Gamma - \mu_2 - v_2^f}{b_2} \\
&\quad - E\left[\min(k_2^f, w_2)\frac{s_1^f + \alpha w_1}{b_2}\right].
\end{aligned}$$

Since $E[w_1] = 0$ and w_1 and w_2 are independent, we obtain

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - E\left[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}\right] \\
&\quad + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f)}{b_2} - \frac{E[\min(k_2^f, w_2)s_1^f]}{b_2} \\
&\quad - \alpha\frac{E[\min(k_2^f, w_2)]E[w_1]}{b_2}.
\end{aligned}$$

Using once more the independence assumption: $E[\min(k_2^f, w_2)s_1^f] = v_2^f E[s_1^f]$. Therefore,

$$\begin{aligned}
E[s_2^f r_2^f] &= E[b_2(r_2^f)^2] - \frac{\mu_2}{b_2}E[s_1^f] - E\left[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}\right] \\
&\quad + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f)}{b_2} - \frac{v_2^f E[s_1^f]}{b_2} \\
&= E[b_2(r_2^f)^2] - E\left[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}\right] \\
&\quad + \frac{(\mu_2 + v_2^f)(\Gamma - \mu_2 - v_2^f - E[s_1^f])}{b_2}.
\end{aligned}$$

Recall that $(\Gamma - \mu_2 - v_2^f - E[s_1^f])/b_2 = E[r_2^f]$. Consequently, $E[s_2^f r_2^f] = E[b_2(r_2^f)^2] - E[(\alpha w_1(s_1^f + \alpha w_1))/b_2] + (\mu_2 + v_2^f)E[r_2^f]$. Computing each term separately,

$$\begin{aligned}
E&\left[\frac{\alpha w_1(s_1^f + \alpha w_1)}{b_2}\right] \\
&= \frac{E[\alpha w_1(b_1 r_1^f + \mu_1 + \min(k_1^f, w_1)) + \alpha^2(w_1)^2]}{b_2} \\
&= \frac{\alpha^2 E[(w_1)^2]}{b_2}\frac{\alpha E[w_1 \min(k_1^f, w_1)]}{b_2}.
\end{aligned}$$

The first term is given by

$$\begin{aligned}
E&[b_2(r_2^f)^2] \\
&= b_2(\mathrm{Var}(r_2^f) + (E[r_2^f])^2) = b_2 E[r_2^f]^2 + b_2\frac{\mathrm{Var}(s_1^f + \alpha w_1)}{b_2^2} \\
&= b_2 E[r_2^f]^2 + \frac{\mathrm{Var}(s_1^f)}{b_2} + \frac{(\alpha)^2\mathrm{Var}(w_1)}{b_2} + \frac{2\,\mathrm{Cov}(s_1^f, \alpha w_1)}{b_2}.
\end{aligned}$$

Note that $\mathrm{Var}(w_1) = E[w_1^2] - E[w_1]^2 = E[w_1^2]$. In addition,

$$\begin{aligned}
\frac{2\,\mathrm{Cov}(s_1^f, \alpha w_1)}{b_2} &= \frac{2\alpha E[w_1 \min(k_1^f, w_1)]}{b_2} \\
&\quad - \frac{2\alpha E[\min(k_1^f, w_1)]E[w_1]}{b_2} = \frac{2\alpha E[w_1 \min(k_1^f, w_1)]}{b_2}.
\end{aligned}$$

Finally, we also have $\mathrm{Var}(s_1^f) = \mathrm{Var}(b_1 r_1^f + \mu_1 + \min(k_1^f, w_1)) = \mathrm{Var}(\min(k_1^f, w_1))$. Therefore,

$$\begin{aligned}
E[s_2^f r_2^f] &= b_2 E[r_2^f]^2 + \frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2} + \frac{(\alpha)^2 E[(w_1)^2]}{b_2} \\
&\quad + \frac{2\alpha E[w_1 \min(k_1^f, w_1)]}{b_2} - \frac{(\alpha)^2 E[(w_1)^2]}{b_2} \\
&\quad - \frac{\alpha E[w_1 \min(k_1^f, w_1)]}{b_2} + (\mu_2 + v_2^f)E[r_2^f].
\end{aligned}$$

As a result, we obtain

$$\begin{aligned}
E[s_2^f r_2^f] &= (b_2 E[r_2^f] + v_2 + \mu_2)E[r_2^f] \\
&\quad + \frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2} + \frac{\alpha E[w_1 \min(k_1^f, w_1)]}{b_2}.
\end{aligned}$$

Using $v_2^f = v_2^c = v_2$, we obtain

$$\begin{aligned}
E[Spending^f] &= \frac{\mathrm{Var}(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 \min(k_1^f, w_1)]}{b_2} \\
&\quad + \frac{-(v_2+\mu_2)^2 b_1^2 - 4\Gamma b_2(v_2+\mu_2)b_1 - 4\Gamma(v_1^f+\mu_1)b_2 b_1 + 4b_1 b_2\Gamma^2}{4b_1 b_2(b_1+b_2)} \\
&\quad + \frac{2(v_1^f+\mu_1)b_2(v_2+\mu_2)b_1 - (v_1^f+\mu_1)^2 b_2^2}{4b_1 b_2(b_1+b_2)}.
\end{aligned}$$

Similarly, for the committed setting we obtain

$$E[Spending^c] = \frac{-(v_2+\mu_2)^2 b_1^2 - 4\Gamma b_2(v_2+\mu_2)b_1 - 4\Gamma(v_1^c+\mu_1)b_2 b_1 + 4b_1 b_2 \Gamma^2}{4b_1 b_2(b_1+b_2)} + \frac{2(v_1^c+\mu_1)b_2(v_2+\mu_2)b_1 - (v_1^c+\mu_1)^2 b_2^2}{4b_1 b_2(b_1+b_2)}.$$

When calculating the difference in spending, we obtain

$$E[Spending^f] - E[Spending^c] = \frac{\mathrm{Var}(\min\{k_1^f, w_1\})}{b_2} + \frac{\alpha E[w_1 \min(k_1^f, w_1)]}{b_2} + \frac{1}{4b_1(b_1+b_2)}[2b_1(v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2(v_1^c+\mu_1)^2 - b_2(v_1^f+\mu_1)^2].$$

Note that $\mathrm{Var}(\min\{k_1^f, w_1\}) > 0$. We also know that from Proposition 1, $v_1^c + \mu_1 > v_1^f + \mu_1 > 0$. Therefore, we get $2b_1 \cdot (v_1^c - v_1^f) > 0$ and $b_2(v_1^c+\mu_1)^2 - b_2(v_1^f+\mu_1)^2 > 0$. The remaining middle term $(2\Gamma - v_2^c - \mu_2)$ is also positive from the assumption that the target is large enough for the subsidy solution to be nontrivial, $\Gamma > E[\epsilon_2]$, which is itself larger than the expected sales quantile, $\Gamma > E[\epsilon_2] > E[\min(k_2^c, \epsilon_2)] > E[\min(k_2^c, w_2)] = v_2^c$. Therefore, $E[Spending^f] > E[Spending^c]$. □

A.4. Proof of Theorem 2

The difference of the variance of sales is given by $\mathrm{Var}(s^c) - \mathrm{Var}(s^f) = E[(s^c)^2 - (s^f)^2] - E[s^c]^2 + E[s^f]^2$. We know that the expected sales are the same in both settings and equal to the target level: $E[s^c]^2 = E[s^f]^2 = \Gamma^2$. Then, we obtain $\mathrm{Var}(s^c) - \mathrm{Var}(s^f) = E[(s^c)^2 - (s^f)^2]$. We now replace the total sales s^c and s^f by the sum at each time period, $s^c = s_1^c + s_2^c$ and $s^f = s_1^f + s_2^f$:

$$\mathrm{Var}(s^c) - \mathrm{Var}(s^f) = E[(s_1^c)^2] + E[(s_2^c)^2] - E[(s_1^f)^2] - E[(s_2^f)^2] + 2E[s_1^c s_2^c - s_1^f s_2^f].$$

By definition, the sales of period 1 under the committed setting are given by

$$s_1^c = \min(x_1 + u_1^c, b_1 r_1^c + \epsilon_1) = \min(b_1 r_1^c + k_1^c + \mu_1, b_1 r_1^c + \mu_1 + w_1) = b_1 r_1^c + \mu_1 + \min(k_1^c, w_1).$$

Then, the second moment can be written as follows:

$$E[(s_1^c)^2] = b_1^2 (r_1^c)^2 + \mu_1^2 + 2b_1 r_1^c(\mu_1 + v_1^c) + 2\mu_1 v_1^c + E[\min(k_1^c, w_1)^2].$$

Similarly, we have $E[(s_1^f)^2] = b_1^2 (r_1^f)^2 + +\mu_1^2 + 2b_1 r_1^f(\mu_1 + v_1^f) + 2\mu_1 v_1^f + E[\min(k_1^f, w_1)^2]$. For s_2^c, the correlation term appears:

$$s_2^c = \min(x_2^c + u_2^c, b_2 r_2^c + \epsilon_2) = b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2).$$

Therefore, $E[(s_2^c)^2] = b_2^2 (r_2^c)^2 + \mu_2^2 + 2b_2 r_2^c(\mu_2 + v_2^c) + 2\mu_2 v_2^c + E[\min(k_2^c, w_2)^2] + \alpha^2 E[w_1^2]$. However, r_2^f is a random variable; therefore, the expectation $E[(s_2^f)^2]$ is calculated differently. We have

$$s_2^f = (\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f) + \mu_2 + \alpha w_1 + \min(k_2^f, w_2) = \Gamma - s_1^f - v_2^f + \min(k_2^f, w_2).$$

Then,

$$(s_2^f)^2 = (\Gamma - v_2^f)^2 + (s_1^f)^2 + (\min(k_2^f, w_2))^2 - 2s_1^f(\Gamma - v_2^f) - 2s_1^f \min(k_2^f, w_2) + 2(\Gamma - v_2^f)\min(k_2^f, w_2).$$

Considering that s_1^f and $\min(k_2^f, w_2)$ are independent, we obtain

$$E[(s_2^f)^2] = (\Gamma - v_2^f)^2 + E[(s_1^f)^2] + E[(\min(k_2^f, w_2))^2] - 2(\Gamma - v_2^f)(E[s_1^f] - v_2^f) - 2(b_1 r_1^f + \mu_1 + v_1^f)v_2^f.$$

We next look at the product: $s_1^c s_2^c = (b_1 r_1^c + \mu_1 + \min(k_1^c, w_1)) \cdot (b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2))$. By expanding the expression, we obtain

$$s_1^c s_2^c = b_1 r_1^c(b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)) + \mu_1(b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)) + \min(k_1^c, w_1)(b_2 r_2^c + \mu_2 + \alpha w_1 + \min(k_2^c, w_2)).$$

Since w_1 and w_2 are independent and $E[w_1] = 0$, we have

$$E[s_1^c s_2^c] = b_1 b_2 r_1^c r_2^c + b_1 r_1^c(\mu_2 + v_2^c) + \mu_1 b_2 r_2^c + \mu_1 \mu_2 + \mu_1 v_2^c + b_2 r_2^c v_1^c + \mu_2 v_1^c + \alpha E[w_1 \min(k_1^c, w_1)] + v_1^c v_2^c.$$

Similarly, we have $s_1^f s_2^f = s_1^f(b_2 r_2^f + \mu_2 + \alpha w_1 + \min(k_2^f, w_2))$. By replacing the expression for r_2^f,

$$s_1^f s_2^f = s_1^f(\Gamma - s_1^f - \mu_2 - \alpha w_1 - v_2^f + \mu_2 + \alpha w_1 + \min(k_2^f, w_2)) = s_1^f(\Gamma - s_1^f - v_2^f + \min(k_2^f, w_2)) = -(s_1^f)^2 + s_1^f(\Gamma - v_2^f) + s_1^f \min(k_2^f, w_2).$$

By again using the independence assumption $E[\min(k_1^f, w_1)\min(k_2^f, w_2)] = v_1^f v_2^f$, we obtain

$$E[s_1^f s_2^f] = -E[(s_1^f)^2] + (\Gamma - v_2^f)(b_1 r_1^f + \mu_1 + v_1^f) + b_1 r_1^f v_2^f \mu_1 v_2^f + v_1^f v_2^f.$$

By simplifying the above expression, $E[s_1^f s_2^f] = \Gamma(b_1 r_1^f + \mu_1 + v_1^f) - E[(s_1^f)^2]$. We now substitute all of the previous expressions in the difference of the variances:

$$\mathrm{Var}(s^c) - \mathrm{Var}(s^f) = (b_1 r_1^c + b_2 r_2^c)^2 + 2(\mu_1 + v_1^c + \mu_2 + v_2^c)(b_1 r_1^c + b_2 r_2^c) + 2(v_1^c + \mu_1)(v_2^c + \mu_2) + (\mu_2 + v_2^c)^2 - \Gamma^2 + \mu_1^2 + 2\mu_1 v_1^c + E[\min(k_1^c, w_1)^2] + \alpha^2 E[w_1^2] + 2\alpha E[w_1 \min(k_1^c, w_1)].$$

Note that $(b_1 r_1^c + \mu_1 + v_1^c) + (b_2 r_2^c + \mu_2 + v_2^c) = E[s_1^c] + E[s_2^c] = \Gamma$. Therefore, we obtain

$$\begin{aligned}
&\mathrm{Var}(s^c) - \mathrm{Var}(s^f) \\
&= \Gamma^2 - (v_1^c)^2 - \Gamma^2 + \mathrm{Var}(\alpha w_1) + E[\min(k_1^c, w_1)^2] + 2\alpha E[w_1 \min(k_1^c, w_1)] \\
&= \mathrm{Var}(\alpha w_1) + (E[\min(k_1^c, w_1)^2] - (v_1^c)^2) + 2\,\mathrm{Cov}(\alpha w_1, \min(k_1^c, w_1)) \\
&= \mathrm{Var}(\alpha w_1) + \mathrm{Var}(\min(k_1^c, w_1)) + 2\,\mathrm{Cov}(\alpha w_1, \min(k_1^c, w_1)) \\
&= \mathrm{Var}(\alpha w_1 + \min(k_1^c, w_1)) \geq 0. \quad \square
\end{aligned}$$

A.5. Proof of Theorem 3

The expected total profits in both settings are given by

$$E[\pi^c] = p_1 E[s_1^c] + p_2 E[s_2^c] - c_1 u_1^c - c_2 E[u_2^c] \quad \text{and}$$
$$E[\pi^f] = p_1 E[s_1^f] + p_2 E[s_2^f] - c_1 u_1^f - c_2 E[u_2^f].$$

By taking the difference, we obtain $E[\pi^f - \pi^c] = p_1 E[s_1^f - s_1^c] + p_2 E[s_2^f - s_2^c] - c_1(u_1^f - u_1^c) - c_2 E[u_2^f - u_2^c]$. By replacing the expressions for the sales and the production quantities, we obtain

$$E[\pi^f - \pi^c] = (p_1 - p_2)(v_1^f - v_1^c)\frac{b_2}{2(b_1+b_2)} + (c_1 - c_2)\left[k_1^c - k_1^f - (v_1^c - v_1^f)\frac{2b_1+b_2}{2(b_1+b_2)}\right].$$

However, we know from the assumption on the profit margins that $0 \le c_1 - c_2 \le p_1 - p_2$ and, therefore,

$$E[\pi^f - \pi^c] \ge (c_1 - c_2)\left[(v_1^f - v_1^c)\frac{b_2}{2(b_1+b_2)} + k_1^c - k_1^f + (v_1^f - v_1^c)\frac{2b_1+b_2}{2(b_1+b_2)}\right].$$

By simplifying the above expression, we have $E[\pi^f - \pi^c] \ge (c_1 - c_2)(v_1^f - v_1^c + k_1^c - k_1^f)$. From Lemma 1, we know that $v_1^f - v_1^c + k_1^c - k_1^f \ge 0$, and we also have, by assumption, $c_1 - c_2 \ge 0$, so that one can conclude $E[\pi^f] \ge E[\pi^c]$. □

A.6. Proof of Theorem 4

1. We have the following expressions for CS_2^f (for simplicity, we focus on the case with $\alpha = 0$):

$$CS_2^f = \frac{(b_2 r_2^f + \epsilon_2)\min(u_2^f + x_2^f, b_2 r_2^f + \epsilon_2)}{2b_2}.$$

We know that $u_2^f + x_2^f = b_2 r_2^f + k_2^f + \mu_2$; therefore,

$$CS_2^f = \frac{(b_2 r_2^f + \epsilon_2)(b_2 r_2^f + \min(k_2^f + \mu_2, \epsilon_2))}{2b_2} \quad \text{and}$$
$$CS_2^c = \frac{(b_2 r_2^c + \epsilon_2)(b_2 r_2^c + \min(k_2^c + \mu_2, \epsilon_2))}{2b_2}.$$

By taking the expectation and computing the difference,

$$E[CS_2^f] - E[CS_2^c] = \frac{b_2(E[(r_2^f)^2] - (r_2^c)^2)}{2} + \frac{E[(r_2^f - r_2^c)(\epsilon_2 + \min(k_2 + \mu_2, \epsilon_2))]}{2}.$$

Here, we used the fact that $k_2^f = k_2^c = k_2$. By using the facts that $\epsilon_2 \ge \min(k_2 + \mu_2, \epsilon_2)$ and $E[(r_2^f)^2] = \mathrm{Var}(r_2^f) + E[(r_2^f)]^2$, we obtain

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] &\ge \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} + E[(r_2^f - r_2^c)(\min(k_2 + \mu_2, \epsilon_2))] \\ &= \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} + E[r_2^f \min(k_2 + \mu_2, \epsilon_2)] - r_2^c(v_2^c + \mu_2). \end{aligned}$$

We know that $r_2^f = (\Gamma - s_1^f - v_2^f - \mu_2)/b_2$; therefore,

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \ge{}& \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} \\ &+ \frac{E[(\Gamma - v_2^f - \mu_2)\min(k_2 + \mu_2, \epsilon_2)]}{b_2} \\ &- \frac{E[(s_1^f)\min(k_2 + \mu_2 + \alpha w_1, \epsilon_2)]}{b_2} \\ &- r_2^c(v_2^c + \mu_2). \end{aligned}$$

By using the facts $v_2^f = v_2^c = v_2$ and $s_1^f = b_1 r_1^f + \mu_1 + \min(k_1^f, w_1)$, we obtain

$$\begin{aligned} &E[CS_2^f] - E[CS_2^c] \\ &\ge \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} + \frac{(v_2 + \mu_2)(\Gamma - v_2 - \mu_2 - b_2 r_2^c)}{b_2} \\ &\quad - \frac{E[b_1 r_1^f \min(k_2 + \mu_2, \epsilon_2) + \mu_1 \min(k_2 + \mu_2, \epsilon_2)]}{b_2} \\ &\quad + \frac{E[\min(k_1^f, w_1)\min(k_2 + \mu_2, \epsilon_2)]}{b_2}. \end{aligned}$$

But $\Gamma - v_2 - \mu_2 - b_2 r_2^c = \Gamma - E[s_2^c] = E[s_1^c]$ and, therefore,

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \ge{}& \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} \\ &+ \frac{(v_2 + \mu_2)E[s_1^c]}{b_2} - \frac{(b_1 r_1^f + v_1^f + \mu_1)(v_2 + \mu_2)}{b_2}. \end{aligned}$$

Now, since $b_1 r_1^f + \mu_1 + v_1^f = E[s_1^f]$, we obtain

$$\begin{aligned} E[CS_2^f] - E[CS_2^c] \ge{}& \frac{b_2(E[(r_2^f)]^2 - (r_2^c)^2)}{2} + \frac{b_2\,\mathrm{Var}(r_2^f)}{2} \\ &+ \frac{(v_2 + \mu_2)(E[s_1^c] - E[s_1^f])}{b_2}. \end{aligned}$$

Note that we have the following inequalities: $E[r_2^f] \ge r_2^c \ge 0$; $\mathrm{Var}(r_2^f) \ge 0$; $E[s_1^c] \ge E[s_1^f]$; showing that $E[CS_2^f] \ge E[CS_2^c]$. □

2. We have the following expression for CS_1^f: $CS_1^f = ((b_1 r_1^f + \epsilon_1)\min(u_1^f + x_1, b_1 r_1^f + \epsilon_1))/(2b_1)$. We know that $u_1^f + x_1 = b_1 r_1^f + k_1^f + \mu_1$ and $\epsilon_1 = \mu_1 + w_1$. Therefore,

$$CS_1^f = \frac{(b_1 r_1^f + \mu_1 + w_1)(b_1 r_1^f + \mu_1 + \min(k_1^f, w_1))}{2b_1} \quad \text{and}$$
$$CS_1^c = \frac{(b_1 r_1^c + \mu_1 + w_1)(b_1 r_1^c + \mu_1 + \min(k_1^c, w_1))}{2b_1}.$$

By taking the expectation and computing the difference,

$$\begin{aligned} &E[CS_1^f] - E[CS_1^c] \\ &= (E[(b_1 r_1^f + \mu_1)^2 - (b_1(r_1^c) + \mu_1)^2 + w_1(b_1 r_1^f - b_1 r_1^c) \\ &\quad + (b_1 r_1^f + \mu_1)\min(k_1^f, w_1)]) \cdot (2b_1)^{-1} \\ &\quad - \frac{E[(b_1 r_1^c + \mu_1)\min(k_1^c, w_1) + w_1(\min(k_1^f, w_1) - \min(k_1^c, w_1))]}{2b_1}. \end{aligned}$$

Since $E[w_1]=0$, we obtain

$$E[CS_1^f]-E[CS_1^c] = \frac{(b_1r_1^f+\mu_1)^2-(b_1r_1^c+\mu_1)^2+(b_1r_1^f+\mu_1)v_1^f-(b_1r_1^c+\mu_1)v_1^c}{2b_1} + \frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}.$$

Note that one can write $r_1^f = r_1^c + (2b_1+b_2)(v_1^c-v_1^f)/(2b_1\cdot(b_1+b_2))$. Therefore, we obtain

$$\begin{aligned}
E[CS_1^f]-E[CS_1^c] &= \frac{1}{2b_1}\left(\left(b_1r_1^c+\mu_1+\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)}\right)^2-(b_1r_1^c+\mu_1)^2\right.\\
&\quad\left.+\left(b_1r_1^c+\mu_1+\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)}\right)v_1^f-(b_1r_1^c+\mu_1)v_1^c\right)\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&= \frac{1}{2b_1}\left((b_1r_1^c+\mu_1)^2-(b_1r_1^c+\mu_1)^2+\left(\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)}\right)^2\right.\\
&\quad\left.+(b_1r_1^c+\mu_1)\frac{(2b_1+b_2)(v_1^c-v_1^f)}{b_1+b_2}\right)\\
&\quad+\frac{1}{2b_1}\left((b_1r_1^c+\mu_1)(v_1^f-v_1^c)+v_1^f\frac{(2b_1+b_2)(v_1^c-v_1^f)}{2(b_1+b_2)}\right.\\
&\quad\left.+E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]\right)\\
&= \frac{1}{2b_1}\left(\left(\frac{(2b_1+b_2)}{2(b_1+b_2)}\right)^2(v_1^c-v_1^f)^2+(v_1^c-v_1^f)\right.\\
&\quad\left.\cdot\left((b_1r_1^c+\mu_1)\frac{b_1}{b_1+b_2}+v_1^f\frac{(2b_1+b_2)}{2(b_1+b_2)}\right)\right)\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}.
\end{aligned}$$

We next rewrite the above expression as a function of the ratio of the price sensitivities $\eta = b_2/b_1$:

$$\begin{aligned}
E[CS_1^f]-E[CS_1^c] &= \frac{v_1^c-v_1^f}{2b_1}\left[(v_1^c-v_1^f)\frac{(2+\eta)^2}{(2(1+\eta))^2}+\frac{b_1r_1^c(\eta)+\mu_1}{1+\eta}+v_1^f\frac{2+\eta}{2(1+\eta)}\right]\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&= \frac{v_1^c-v_1^f}{2b_1(1+\eta)}\left[(v_1^c-v_1^f)\frac{(2+\eta)^2}{4(1+\eta)}+b_1r_1^c(\eta)+\mu_1+v_1^f\frac{2+\eta}{2}\right]\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}\\
&= \frac{v_1^c-v_1^f}{2b_1(1+\eta)}\left[\frac{(1+\eta/2)^2}{1+\eta}(v_1^c-v_1^f)+b_1r_1^c(\eta)+\mu_1+v_1^f\left(1+\frac{\eta}{2}\right)\right]\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}.
\end{aligned}$$

We next show that $E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]$ is always nonpositive. Since $k_1^f \le k_1^c$, we have

$$w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1)) = 0 \qquad \text{when } w_1 \le k_1^f,$$

$$w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1)) = w_1(k_1^f-k_1^c) \le 0 \qquad \text{when } w_1 \ge k_1^c,$$

$$w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1)) \le w_1(k_1^f-w_1) \le 0 \qquad \text{when } k_1^f \le w_1 \le k_1^c.$$

Therefore, $E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]\le 0$. We next study the limits of $E[CS_1^f]-E[CS_1^c]$ when η goes to zero and when η goes to infinity. Without loss of generality, we assume that b_1 is a given constant and that b_2 is varying. Note that

$$b_1r_1^c(\eta) = \frac{\Gamma-(v_2+\mu_2)/2-(v_1^c+\mu_1)(1+\eta/2)}{1+\eta}.$$

By taking the limit, we obtain $\lim_{\eta\to+\infty} b_1r_1^c(\eta) = -(v_1^c+\mu_1)/2$. As a result,

$$\lim_{\eta\to+\infty} E[CS_1^f]-E[CS_1^c] = \frac{(v_1^c-v_1^f)(v_1^c+v_1^f)+4E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{8b_1} \le 0.$$

We next study the limit when $\eta \to 0$:

$$\lim_{\eta\to 0} E[CS_1^f]-E[CS_1^c] = \frac{1}{2b_1}E\left[(\min(k_1^c,w_1)-\min(k_1^f,w_1))\left(\Gamma-\frac{v_2+\mu_2}{2}-w_1\right)\right].$$

We know that $\Gamma \ge 2w_1$. In addition, from Assumption 3.3, $\Gamma \ge 2(v_2+\mu_2)$. Therefore, we obtain $\Gamma-(v_2+\mu_2)/2-w_1 \ge 0$ and $\lim_{\eta\to 0} E[CS_1^f]-E[CS_1^c] \ge 0$. Consequently, we have shown that when the ratio of price sensitivities approaches zero, the consumers are better off in the flexible setting, whereas when this ratio approaches infinity, the consumers are better off in the committed setting, in terms of expected consumer surplus. To conclude the proof, we next show that the difference in expected consumer surplus, $E[CS_1^f]-E[CS_1^c]$, is nonincreasing with η. Recall that we have

$$\begin{aligned}
E[CS_1^f]-E[CS_1^c] &= \frac{v_1^c-v_1^f}{2b_1}\frac{1}{1+\eta}\left(\frac{(1+\eta/2)^2}{1+\eta}(v_1^c-v_1^f)+b_1r_1^c(\eta)+\mu_1+v_1^f\left(1+\frac{\eta}{2}\right)\right)\\
&\quad+\frac{E[w_1(\min(k_1^f,w_1)-\min(k_1^c,w_1))]}{2b_1}.
\end{aligned}$$

Note that the second term does not depend on η. The derivative is given by

$$\begin{aligned}
\frac{\partial(E[CS_1^f]-E[CS_1^c])}{\partial\eta} &= \frac{v_1^c-v_1^f}{2b_1}\\
&\quad\cdot\left[\frac{(1+\eta)(v_1^f/2+(v_1^c-v_1^f)(\eta(1+\eta/2)/(2(1+\eta)^2))+b_1(\partial r_1^c/\partial\eta))}{(1+\eta)^2}\right.\\
&\quad\left.-\frac{(v_1^c-v_1^f)((1+\eta/2)^2/(1+\eta))+v_1^f(1+\eta/2)+\mu_1+b_1r_1^c(\eta)}{(1+\eta)^2}\right].
\end{aligned}$$

By rearranging and simplifying, we obtain

$$\begin{aligned}
\frac{\partial(E[CS_1^f]-E[CS_1^c])}{\partial\eta} &= \frac{v_1^c-v_1^f}{2b_1(1+\eta)^2}\left[(v_1^c-v_1^f)\left[\frac{\eta(1+\eta/2)}{2(1+\eta)}-\frac{(1+\eta/2)^2}{1+\eta}\right]\right.\\
&\quad\left.+(1+\eta)b_1\frac{\partial r_1^c}{\partial\eta}-b_1r_1^c(\eta)-\mu_1+v_1^f\left(\frac{1+\eta}{2}-\frac{2+\eta}{2}\right)\right].
\end{aligned}$$

Note that we have

$$b_1 r_1^c(\eta) = \frac{\Gamma - (v_2+\mu_2)/2 - (v_1^c+\mu_1)(1+\eta/2)}{1+\eta} \quad \text{and}$$
$$\frac{b_1 \partial r_1^c}{\partial \eta} = \frac{-\Gamma + (v_2+\mu_2)/2 + (v_1^c+\mu_1)/2}{(1+\eta)^2}.$$

Therefore,

$$\begin{aligned}
&\frac{\partial(E[CS_1^f]-E[CS_1^c])}{\partial\eta}\\
&= \frac{v_1^c - v_1^f}{2b_1(1+\eta)^2}\left[-\mu_1 - \frac{v_1^f}{2} - (v_1^c - v_1^f)\frac{1+\eta/2}{1+\eta}\right.\\
&\qquad \left.+ \frac{-2\Gamma + (v_2+\mu_2) + (v_1^c+\mu_1)((3+\eta)/2)}{1+\eta}\right]\\
&= \frac{v_1^c - v_1^f}{2b_1(1+\eta)^2}\left[\mu_1 \frac{(3+\eta)/2 - (1+\eta)}{1+\eta} + v_1^f\left(\frac{1}{2} + \frac{1}{2(1+\eta)} - \frac{1}{2}\right)\right.\\
&\qquad \left.+ v_1^c \frac{(3+\eta)/2 - 1 - \eta/2}{1+\eta} + \frac{-2\Gamma + (v_2+\mu_2)}{1+\eta}\right]\\
&= \frac{v_1^c - v_1^f}{2b_1(1+\eta)^3}\left[\mu_1 \frac{1-\eta}{2} + \frac{v_1^f}{2} + \frac{v_1^c}{2} + (-2\Gamma + (v_2+\mu_2))\right]\\
&= \frac{v_1^c - v_1^f}{2b_1(1+\eta)^3}\left[\frac{v_1^f}{2} - \frac{\eta\mu_1}{2} + \left(-2\Gamma + (v_2+\mu_2) + \frac{\mu_1 + v_1^c}{2}\right)\right].
\end{aligned}$$

Note that we have the following inequalities: $v_1^c - v_1^f \geq 0$; $b_1 \geq 0$; $v_1^f \leq 0$; $\eta\mu_1 \geq 0$; $\Gamma \geq 2(v_2+\mu_2)$; $\Gamma \geq 2(v_1^c+\mu_1)$. Therefore, we obtain $\partial(E[CS_1^f] - E[CS_1^c])/\partial\eta \leq 0$. As a result, there exists a threshold value of b_1/b_2 above (below) which the consumers are better off in the flexible (committed) setting. □

A.7. Extending the Result from Lippman and McCardle (1997) to Two Time Periods

In Lippman and McCardle (1997), the authors show the equivalence between a monopolist and a setting with several competing firms, for a single-period setting. One can use the work in Caro and Martínez-de Albéniz (2010) to extend the result to the case with two periods. The result is shown for two firms, but one can use a similar methodology and extend the analysis for $N \geq 2$ suppliers. Consider two suppliers that sell substitutable products, and each firm maximizes the total expected profits (using a lost-sales model) over a two-period horizon. The necessary assumptions are as follows.

1. The aggregate customer demand D_t in period $t = 1, 2$ is continuous, stochastic and may be correlated across periods. Define q^i as the proportion of the total demand allocated initially to firms $i = 1, 2$. The demand split is assumed to be stationary over time and $q^i + q^j = 1$.
2. The effective demand, R_t^i, faced by supplier i at time t is composed of the original demand and the spillover demand from the competing supplier j: $R_t^i = q^i D_t + (q^j D_t - (u_t^j + x_t^j))^+$.
3. The unit cost and price for each firm in period t are exogenous and symmetric, denoted by c_t and p_t, respectively.
4. Leftover inventory can be carried over from the first to the second period and is lost at the end of the season. If both firms stock out in a given period, the unsatisfied demand is lost as well.

These assumptions are derived from similar assumptions used in Theorem 3 of Lippman and McCardle (1997) and Theorem 4 of Caro and Martínez-de Albéniz (2010) (presented only in their appendix). Under these conditions, the firms observe an aggregate industry-wide shock. Therefore, the two suppliers will either stock out together or have excess inventory together. Note that the result is proved for a setting with suppliers that have symmetric prices and costs. Nevertheless, it still allows the firms to have different initial inventories and to receive unequal shares of the uncertain demand—i.e., $q^i \neq q^j$.

A.8. Proof of Proposition 3

The committed production quantile can be derived in a similar manner as in Caro and Martínez-de Albéniz (2010). Indeed, in the committed setting, the government decides the rebates up front and then the same game is played among the suppliers as in Caro and Martínez-de Albéniz (2010). The main intuition behind their proof is as follows: assuming that both suppliers are symmetric, they will either have leftover inventory or will stock out simultaneously. This is derived from the common demand shock assumption that is shared by both firms, leading both suppliers to experience identical outcomes. Therefore, the demand spillage between firms never occurs in the equilibrium path, and both firms produce (in aggregate) the same as the monopolist production quantity.

We next consider the flexible setting. We note that the first-period production quantile in the flexible setting is more complicated to derive. In the second period, the firms will produce $q^i(b_2 r_2 + \mu_2 + k_2)$, as in the committed setting, where q^i is firm i's share of the demand. The optimal second-period subsidy is the same as in the monopolistic case, since the combined sales from both suppliers is the same, $s_1 = s_1^i + s_1^j$:

$$r_{2f}^*(s_1) = \frac{\Gamma - s_1 - v_2 - \mu_2}{b_2}.$$

In the first period, supplier i will optimize his profits using the following revenue-to-go function: $h_2^{f_i} = p_2 E[\min(q^i(b_2 r_2^f + \mu_2 + k_2), R_2^i)] - c_2[q^i(b_2 r_2^f + \mu_2 + k_2) - x_2^i]$. In this case, R_2^i represents the realized demand for firm i after considering the residual demand from firm j—that is, $R_2^i = q^i D_2^i + (q^j D_2^j - u_2^j - x_2^j)^+$. Replacing the optimal second-period subsidy, we obtain

$$h_2^{f_i} = p_2 q^i(\Gamma - s_1) - c_2[q^i(\Gamma - s_1 - v_2 + k_2) - x_2^i].$$

Note that $x_2^i = u_2^i - s_1^i$. Taking the derivative of $h_2^{f_i}$ with respect to u_1^i yields

$$\frac{dh_2^{f_i}}{du_1^i} = -p_2 q^i \frac{ds_1}{du_1^i} + c_2 q^i \frac{ds_1}{du_1^i} + c_2\left(1 - \frac{ds_1^i}{du_1^i}\right).$$

We have $s_1 = \min(u_2^i + u_2^j, D_1)$ and $s_1^i = \min(u_1^i, R_1^i)$, where R_1^i is the realized demand of firm i. Therefore,

$$\begin{aligned}
\frac{dh_2^{f_i}}{du_1^i} &= -p_2 q^i P(u_2^i + u_2^j \leq D_1) + c_2 q^i P(u_2^i + u_2^j \leq D_1)\\
&\quad + c_2[1 - P(u_1^i \leq R_1^i)].
\end{aligned}$$

The first-order condition for supplier i in the first period is then given by

$$p_1 P(u_1^i \leq R_1^i) - c_1 + \frac{dh_2^{f_i}}{du_1^i} = 0.$$

Using a similar argument as in Caro and Martínez-de Albéniz (2010) and as in our committed setting, assuming that the firms are symmetric, it can be shown that in equilibrium, both firms will either stock out or have excess supply simultaneously. Consequently, we simplify the notation with $\bar{F} = P(u_1^i \le R_1^i) = P(u_2^i + u_2^j \le D_1)$, where $\bar{F}$ is the complementary cdf of the realized demand distribution R_1^i. As a result, the first-order condition reduces to

$$p_1\bar{F} - c_1 - p_2 q^i \bar{F} + c_2 q^i \bar{F} + c_2(1-\bar{F}) = 0,$$

where we used the fact that $ds_1/du_1^i = \bar{F}$. This equation simplifies to

$$\bar{F} = \frac{c_1 - c_2}{p_1 - p_2 q^i + c_2 q^i - c_2}.$$

Therefore, the resulting quantile is given by

$$k_1^{f^i} = F^{-1}(1-\bar{F}) = F^{-1}\left(\frac{(p_1 - c_1) - q^i(p_2 - c_2)}{(p_1 - c_2) - q^i(p_2 - c_2)}\right).$$

Finally, the inequality $k_1^{f^i} \le k_1^c$ is easy to show, by noting that the derivative of the quantile $k_1^{f_i}$ with respect to q^i is negative, together with the boundary conditions at 0 and 1. This concludes the proof of Proposition 3.

A.9. Proof of Proposition 6

Our goal is to show the inequality for both the expected spending and the variance of the sales. We first need to undertake some preliminary calculations on the optimal quantity and rebate levels.

Preliminary Calculations. We next derive all of the necessary expressions so as to compute the expected spending and the variance of the sales.

We start by the production quantity in the second period. Observe that in the semiflexible setting, the supplier's problem remains exactly the same as in the flexible setting. We thus obtain the same expression for the second-period production quantity:

$$u_2^s = b_2 r_2^s + \mu_2 + k_2^s - x_2.$$

In addition, note that $k_2^s = k_2^f = k_2^c$. We next compute the rebate in the second period. As in the flexible setting, we start by solving the second-period government's problem:

$$\begin{aligned}\min_{r_2}\ & E_{\epsilon_1,\epsilon_2}[r_2(s_1).s_2(r_2(s_1),\epsilon_2)]\\ \text{s.t.}\ & E_{\epsilon_1}[s_1] + E_{\epsilon_1,\epsilon_2}[s_2(r_2(s_1),\epsilon_2] \ge \Gamma\end{aligned}$$

We know that $s_2 = b_2 r_2 + \mu_2 + \min(k_2, w_2)$. Since ϵ_1 and ϵ_2 are assumed to be independent, r_2 and s_2 are also independent. Therefore, the government's problem in the second period becomes

$$\begin{aligned}\min_{r_2}\ & E_{\epsilon_1}[r_2(s_1)\cdot(b_2 r_2(s_1) + \mu_2 + v_2)]\\ \text{s.t.}\ & E_{\epsilon_1}[s_1] + E_{\epsilon_1}[b_2 r_2(s_1)] + \mu_2 + v_2 \ge \Gamma.\end{aligned}$$

Note that in the flexible setting, s_1 is already realized at this point. However, in the semiflexible setting, we do not know the realization of s_1. To make the analysis tractable, we assume that s_1 is an n-points distribution, with $n \in \mathbb{N}^*$. More precisely, s_1^i has a probability q_i of being realized (such that $\sum_{i=1}^n q_i = 1$), and r_2^i is the rebate associated to the realization s_1^i.
The problem becomes

$$\begin{aligned}\min_{r_2^i}\ & \sum_{i=1}^n [q_i r_2^i(b_2 r_2^i + \mu_2 + v_2)]\\ \text{s.t.}\ & \sum_{i=1}^n [q_i(b_2 r_2^i + \mu_2 + v_2)] \ge \Gamma - E_{\epsilon_1}[s_1].\end{aligned}$$

We derive the optimal solution of this problem at the end of this proof (see below). The solution is given by

$$\forall\, i \in \{1,\dots,n\}\colon r_2^i = \frac{\Gamma - E[s_1] - \mu_2 - v_2}{b_2}.$$

In other words, for each $i = 1,\dots,n$, r_2^i is the same and does not depend on the probability q_i. Therefore, for any (discrete) distribution of s_1, we have the following result:

$$r_2^s = \frac{\Gamma - E[s_1] - \mu_2 - v_2}{b_2}.$$

In particular, in the semiflexible setting, r_2 does not depend on s_1 but only on $E[s_1]$. Note that this expression is very close to the one we found for the rebate in the flexible setting:

$$r_2^f = \frac{\Gamma - s_1^f - \mu_2 - v_2}{b_2}.$$

Finally, we proceed to compute the production quantity and the first-period subsidy level. Note that the difference between r_2 in the flexible and the semiflexible settings does not affect these expressions. We then obtain the exact same results as in the flexible setting:

$$u_1^s = u_1^f \quad \text{and} \quad r_1^s = r_1^f;$$

therefore,

$$r_2^s = E[r_2^f] \quad \text{and} \quad u_2^s = E[u_2^f].$$

As a result, the semiflexible setting admits very similar results as the flexible policy. Nevertheless, these little differences have an impact on the overall government spending, and on the variance of the sales, as we will see next.

Expected Spending. We first compare the expected spending of the semiflexible setting relative to the flexible policy. We have

$$E[Sp^f - Sp^s] = r_1^f E[s_1^f] - r_1^s E[s_1^s] + E[r_2^f s_2^f] - E[r_2^s s_2^s].$$

Since $r_1^f = r_1^s$ and $s_1^f = s_1^s$, we obtain $E[Sp^f - Sp^s] = E[r_2^f s_2^f] - E[r_2^s s_2^s]$. From our previous calculations, we know that

$$E[r_2^f s_2^f] = (b_2 E[r_2^f] + \mu_2 + v_2)E[r_2^f] + \frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2}.$$

We next compute the second term:

$$\begin{aligned}E[r_2^s s_2^s] &= E[r_2^s(b_2 r_2^s + \mu_2 + \min(k_2, w_2)]\\ &= b_2 E[(r_2^s)^2] + \mu_2 E[r_2^s] + E[\min(k_2, w_2) r_2^s].\end{aligned}$$

Note that $r_2^s = E[r_2^f]$ and, therefore,

$$\begin{aligned}E[r_2^s s_2^s] &= b_2 E[r_2^f]^2 + \mu_2 E[r_2^f] + v_2 E[r_2^f]\\ &= (b_2 E[r_2^f] + \mu_2 + v_2)E[r_2^f].\end{aligned}$$

The overall difference in expected spending can then be written as

$$E[Sp^f - Sp^s] = \frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2} \geq 0.$$

We next compare the semiflexible and the committed settings. We have

$$E[Sp^s - Sp^c] = E[Sp^s] - E[Sp^f] + E[Sp^f] - E[Sp^c]$$

$$E[Sp^s - Sp^c] = -\frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2} + \frac{\mathrm{Var}(\min(k_1^f, w_1))}{b_2} + \frac{1}{4b_1(b_1+b_2)}[2b_1(v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2(v_1^c + \mu_1)^2 - b_2(v_1^f + \mu_1)^2]$$

$$E[Sp^s - Sp^c] = \frac{1}{4b_1(b_1+b_2)}[2b_1(v_1^c - v_1^f)(2\Gamma - v_2^c - \mu_2) + b_2(v_1^c + \mu_1)^2 - b_2(v_1^f + \mu_1)^2].$$

We have shown in the paper that this expression is positive and, therefore, $E[Sp^s] \geq E[Sp^c]$. In conclusion, we have

$$E[Sp^f] \geq E[Sp^s] \geq E[Sp^c].$$

Variance of the Sales. As before, we start by comparing the semiflexible and the flexible settings. We have

$$\mathrm{Var}(s^f) - \mathrm{Var}(s^s) = E[(s_1^f)^2] + E[(s_2^f)^2] - E[(s_1^s)^2] - E[(s_2^s)^2] + 2E[s_1^f s_2^f - s_1^s s_2^s].$$

Since $s_1^f = s_1^s$, we obtain

$$\mathrm{Var}(s^f) - \mathrm{Var}(s^s) = E[(s_2^f)^2] - E[(s_2^s)^2] + 2E[s_1^f(s_2^f - s_2^s)]. \quad \text{(A.11)}$$

We next compute the different terms separately. We know that $s_2^f = b_2 r_2^f + \mu_2 + \min(k_2, w_2)$ and $s_2^s = b_2 r_2^s + \mu_2 + \min(k_2, w_2)$. Therefore, we obtain

$$E[(s_2^f)^2] - E[(s_2^s)^2] = b_2^2 E[(r_2^f)^2 - (r_2^s)^2] + 2b_2\mu_2 E[r_2^f - r_2^s] + 2b_2\mu_2 E[\min(k_2, w_2)(r_2^f - r_2^s)].$$

In addition, we have

$$r_2^f - r_2^s = \frac{\Gamma - s_1^f - \mu_2 - v_2 - \Gamma + E[s_1^s] + \mu_2 + v_2}{b_2} = \frac{E[s_1^s] - s_1^f}{b_2}.$$

We next use the fact that the first-period sales are equal in both settings:

$$r_2^f - r_2^s = \frac{E[s_1^f] - s_1^f}{b_2} = \frac{v_1^f - \min(k_1^f, w_1)}{b_2}.$$

As a result, $2b_2\mu_2 E[r_2^f - r_2^s] = 0$. Since w_1 and w_2 are assumed to be independent, we also have $2b_2\mu_2 E[\min(k_2, w_2) \cdot (r_2^f - r_2^s)] = 0$. Then, we have

$$E[(s_2^f)^2] - E[(s_2^s)^2] = b_2^2 E[(r_2^f)^2 - (r_2^s)^2]$$

$$E[(s_2^f)^2] - E[(s_2^s)^2] = b_2^2 E\left[\left(\frac{\Gamma - s_1^f - \mu_2 - v_2}{b_2}\right)^2 - \left(\frac{\Gamma - E[s_1^s] - \mu_2 - v_2}{b_2}\right)^2\right]$$

$$E[(s_2^f)^2] - E[(s_2^s)^2] = E[(\Gamma - \mu_2 - v_2)^2 + (s_1^f)^2 - 2s_1^f(\Gamma - \mu_2 - v_2) - (\Gamma - \mu_2 - v_2)^2 - E[s_1^s]^2 + 2E[s_1^s](\Gamma - \mu_2 - v_2)]$$

$$E[(s_2^f)^2] - E[(s_2^s)^2] = E[(s_1^f)^2 - E[s_1^s]^2 + 2(\Gamma - \mu_2 - v_2)(E[s_1^s] - s_1^f)].$$

Since $s_1^f = s_1^s$, we obtain

$$E[(s_2^f)^2] - E[(s_2^s)^2] = E[(s_1^f)^2 - E[s_1^f]^2] = \mathrm{Var}(s_1^f) = \mathrm{Var}(\min(k_1^f, w_1)).$$

We next compute the last term in Equation (A.11). We have

$$2E[s_1^f(s_2^f - s_2^s)] = 2E[s_1^f b_2(r_2^f - r_2^s)] = 2E[s_1^f(v_1^f - \min(k_1^f, w_1))]$$

$$2E[s_1^f(s_2^f - s_2^s)] = 2E[(b_1 r_1^f + \mu_1 + \min(k_1^f, w_1))(v_1^f - \min(k_1^f, w_1))]$$

$$2E[s_1^f(s_2^f - s_2^s)] = 2(v_1^f)^2 - E[\min(k_1^f, w_1)^2] = -2\,\mathrm{Var}(\min(k_1^f, w_1)).$$

Therefore,

$$\mathrm{Var}(s^f) - \mathrm{Var}(s^s) = \mathrm{Var}(\min(k_1^f, w_1)) - 2\,\mathrm{Var}(\min(k_1^f, w_1)) = -\mathrm{Var}(\min(k_1^f, w_1)) \leq 0,$$

concluding this part of the proof.

Finally, we present the proof for the comparison between the committed and semiflexible settings. We have

$$\mathrm{Var}(s^c) - \mathrm{Var}(s^s) = \mathrm{Var}(s^c) - \mathrm{Var}(s^f) + \mathrm{Var}(s^f) - \mathrm{Var}(s^s).$$

We know from Theorem 2 that $\mathrm{Var}(s^c) - \mathrm{Var}(s^f) = \mathrm{Var}(\min(k_1^c, w_1))$, and we just showed that $\mathrm{Var}(s^f) - \mathrm{Var}(s^s) = -\mathrm{Var}(\min(k_1^f, w_1))$. Therefore,

$$\mathrm{Var}(s^c) - \mathrm{Var}(s^s) = \mathrm{Var}(\min(k_1^c, w_1)) - \mathrm{Var}(\min(k_1^f, w_1)).$$

Since $k_1^c \geq k_1^f$, we can conclude $\mathrm{Var}(s^c) \geq \mathrm{Var}(s^s)$.

Proof of the Optimal Semiflexible Subsidy Policy. Consider solving the following problem:

$$\min_{r_2^i} \sum_{i=1}^{n} [q_i r_2^i (b_2 r_2^i + \mu_2 + v_2)]$$
$$\text{s.t.} \sum_{i=1}^{n} [q_i (b_2 r_2^i + \mu_2 + v_2)] \geq \Gamma - E_{\epsilon_1}[s_1].$$

We denote the objective function by J_n. As in the flexible setting, the objective function and the constraint are nondecreasing with respect to r_2^i. Consequently, the optimal solution is obtained when the constraint is exactly met. Therefore, we can express r_2^n as a function of r_2^i for $i = 1, \ldots, n-1$:

$$r_2^n = \frac{\Gamma - E[s_1] - \sum_{i=1}^{n-1}[q_i(b_2 r_2^i + \mu_2 + v_2)]}{b_2 q_n} - \frac{\mu_2 + v_2}{b_2}.$$

We now insert this expression in the objective function:

$$J_n = \sum_{i=1}^{n-1}[q_i r_2^i(b_2 r_2^i + \mu_2 + v_2)] + \left[\frac{\Gamma - E[s_1] - \sum_{i=1}^{n-1}(q_i(b_2 r_2^i + \mu_2 + v_2))}{b_2} - q_n\frac{\mu_2 + v_2}{b_2}\right] \cdot \left[\frac{\Gamma - E[s_1] - \sum_{i=1}^{n-1}(q_i(b_2 r_2^i + \mu_2 + v_2))}{q_n}\right]$$

$$J_n = \sum_{i=1}^{n-1}[q_i r_2^i(b_2 r_2^i + \mu_2 + v_2)] + \frac{\{\Gamma - E[s_1] - \sum_{i=1}^{n-1}[q_i(b_2 r_2^i + \mu_2 + v_2)]\}^2}{b_2 q_n} + \frac{\sum_{i=1}^{n-1}[q_i(b_2 r_2^i + \mu_2 + v_2)](\mu_2 + v_2)}{b_2} - \frac{(\mu_2 + v_2)(\Gamma - E[s_1])}{b_2}.$$

Note that the objective is the sum of convex functions and, hence, convex. Therefore, the first-order condition is sufficient and necessary to find the global minimum.

We set $i \in \{1, \ldots, n\}$, and differentiate J_n with respect to r_2^i:

$$\frac{\partial J_n}{\partial r_2^i} = 2b_2 q_i r_2^i + 2q_i(\mu_2 + v_2) + 2\frac{[\Gamma - E[s_1] - \sum_{j=1}^{n-1}(q_j(b_2 r_2^j + \mu_2 + v_2))](-b_2 q_i)}{b_2 q_n}$$

$$\frac{\partial J_n}{\partial r_2^i} = 2b_2 q_i r_2^i + 2q_i(\mu_2 + v_2) - 2\frac{q_i}{q_n}(\Gamma - E[s_1]) + 2\frac{q_i}{q_n}\sum_{j=1}^{n-1}(q_j(b_2 r_2^j + \mu_2 + v_2)))$$

$$\frac{\partial J_n}{\partial r_2^i} = r_2^i\left(2b_2 q_i + \frac{2b_2 q_i^2}{q_n}\right) + 2q_i(\mu_2 + v_2) - 2\frac{q_i}{q_n}(\Gamma - E[s_1]) + \frac{2q_i}{q_n}\left[\sum_{j=1}^{i-1}[q_j(b_2 r_2^j + \mu_2 + v_2)] + \sum_{j=i+1}^{n-1}[q_j(b_2 r_2^j + \mu_2 + v_2)]\right] + \frac{2q_i}{q_n}[q_i(\mu_2 + v_2)].$$

We next solve the equation $\partial J_n / \partial r_2^i = 0$. By multiplying both sides of the equation by $q_n / 2b_2 q_i$ and merging some terms, we obtain

$$r_2^i(q_n + q_i) - \bar{r}_2 + \frac{1}{b_2}\left[\sum_{j=1}^{i-1} q_j(b_2 r_2^j) + \sum_{j=i+1}^{n-1} q_j(b_2 r_2^j)\right] = 0, \quad \text{(A.12)}$$

where $\bar{r}_2 = (\Gamma - E[s_1] - \mu_2 - v_2)/b_2$. We next subtract the equation with r_2^i from the one with r_2^{i+1}:

$$0 = r_2^{i+1}(q_n + q_{i+1}) - \bar{r}_2 + \frac{1}{b_2}\left[\sum_{j=1}^{i} q_j(b_2 r_2^j) + \sum_{j=i+2}^{n-1} q_j(b_2 r_2^j) - r_2^i(q_n + q_i)\right] + \bar{r}_2 - \frac{1}{b_2}\left[\sum_{j=1}^{i-1} q_j(b_2 r_2^j) + \sum_{j=i+1}^{n-1} q_j(b_2 r_2^j)\right] = q_n(r_2^{i+1} - r_2^i)$$

$$+ q_{i+1} r_2^{i+1} - q_i r_2^i + \frac{1}{b_2}(q_i b_2 r_2^i - q_{i+1} b_2 r_2^{i+1}) = q_n(r_2^{i+1} - r_2^i).$$

Since $q_n \neq 0$, we obtain $r_2^i = r_2^{i+1}$. In other words, $r_2^i = r_2$ for all $i = 1, \ldots, n$. We next consider Equation (A.12) and isolate r_2:

$$0 = r_2(q_n + q_i) - \bar{r}_2 + \frac{1}{b_2}\left[\sum_{j=1}^{i-1} q_j(b_2 r_2) + \sum_{j=i+1}^{n-1} q_j(b_2 r_2)\right].$$

Therefore, we obtain $r_2 \sum_{j=1}^{n} q_j = \bar{r}_2$. Recall that $\sum_{j=1}^{n} q_j = 1$, so that $r_2 = \bar{r}_2$.

A.10. Description of Data in Computational Experiments

The price of an installation was based on the average installation price of Q1 and Q2 of 2011, $p_1 = 2.546$€/W and $p_2 = 2.42$€/W, respectively.[8] We used a cost of installation roughly at 80% of the final price,[9] $c_1 = 2.03$€/W. We vary the second-period cost to display the intertemporal difference in profit margins. In particular, we use values of c_2 ranging between 1.906 and 2.03€/W.

This range of values explores the nontrivial regime we discuss in this paper. The lower bound is imposed by $p_1 - c_1 > p_2 - c_2$; otherwise, the supplier would delay all of its production to the second period when facing a flexible government. The upper bound is due to the condition that $c_1 > c_2$; otherwise, most of the second-period supply would be produced within the first period. Salvage value at the end of the horizon is set at $p_3 = 1.8$€/W, which does not affect the qualitative aspect of the simulation. Note that we must use a salvage value lower than the cost in period 2, c_2; otherwise, the problem becomes trivial with a direct incentive to oversupply.

Given the total number of installations equal to 3,806 MW in 2009 and 7,400 MW in 2010, we use the price and rebate level to estimate a simple linear sensitivity to rebate levels. Prices of solar panels at the time were 3.9€/W and 2.8€/W, respectively, for 2009 and 2010. The feed-in-tariff level was 0.43€/kWh in 2009 and between 0.33 and 0.39€/kWh in 2010. These tariffs reflect the sale price of electricity generated from the solar panel, which is fixed for 20 years from the installation of the solar panel. Considering the average annual output of solar panels in Germany (876 kWh/kW) and the resulting 20-year stream of cash flows discounted at 5% minus the upfront cost, we obtain a net present value of an installation at 0.79€/W and 1.13€/W in 2009 and 2010, respectively. Evaluating the rate of increased demand based on the increased economic benefit of a solar panel, we obtain $b = (7{,}400 - 3{,}806)/(1.13 - 0.79) = 10{,}571$. In other words, for every €/W of subsidy, we expect to obtain an additional 10,571 MW of installations. In this section, we assume this sensitivity b to be the same over time. Using as a base the electricity price of 0.25€/kWh instead of the feed-in-tariff, we estimate the nominal demand for solar panels in the first and second half of 2011 at $\mu_1 = 1{,}839$ MW and $\mu_2 = 3{,}150$ MW. Considering a target adoption of $\Gamma = 7{,}500$ MW, we use our model to find the optimal subsidy (feed-in-tariff) and the industry's supply level. For comparison purposes, the historical value of the feed-in-tariff in 2011 was 0.2874€/kWh. We use 0.25€/kWh to be a baseline feed-in-tariff, which would lead to the nominal levels of demand μ_1 and μ_2. The optimal feed-in-tariff recommended by our model, depending on the demand uncertainty and costs, is in the range $[0.281, 0.287]$€/kWh for the committed setting and $[0.281, 0.292]$€/kWh for the flexible setting.

In the first set of simulations, Figures 3 and 4, we vary both the second-period cost of production, $c_2 \in [1.906, 2.03]$€/W, and the magnitude of the demand uncertainty w_1 and w_2. We draw both w_1 and w_2 independently from a uniform distribution, ranging from $-A$ to A. Starting at a low value of $A = 10$ MW and increasing it to $A = 920$ MW, we emulate various levels of standard deviation of demand uncertainty, σ. We restrict our simulation to values of A smaller than the average nominal demand μ, therefore preventing negative demands.

Endnotes

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